# Small Business Cost of Capital

## A Report for AIGT

## Version 4.f

Prepared by

London Economics

October 2010

## About London Economics

London Economics is one of Europe's leading specialist economics and policy consultancies and has its head office in London. We also have offices in Brussels, Dublin, Cardiff and Budapest, and associated offices in Paris and Valletta.

We advise clients in both the public and private sectors on economic and financial analysis, policy development and evaluation, business strategy, and regulatory and competition policy. Our consultants are highly-qualified economists with experience in applying a wide variety of analytical techniques to assist our work, including cost-benefit analysis, multi-criteria analysis, policy simulation, scenario building, econometric and statistical analysis and mathematical modelling. We are also experienced in using a wide range of data collection techniques including literature reviews, survey questionnaires, interviews and focus groups.

Head Office: 11-15 Betterton Street, London, WC2H 9BP, United Kingdom.

w: www.londecon.co.uk e: info@londecon.co.uk

t: +44 (0)20 7866 8185 f: +44 (0)20 7866 8186

Wherever possible London Economics uses paper sourced from sustainably managed forests using production processes that meet the EU eco-label requirements.

***Contents*** ***Page***

# Tables, Figures & Boxes

# Tables, Figures & Boxes

*Page*

# Executive Summary

Ofgem is consulting on the cost of capital for the legacy IGT sector in the UK as part of their normal 5-year review.

London Economics has estimated the weighted average cost of capital (WACC) for small companies in the IGT sector. We have only estimated new WACC parameters for the small company debt premium and the small company equity premium, while the other WACC parameters, we base on previous regulatory decisions; we take the midpoint(s) from the previous CC and Ofgem Decisions' ranges for our estimates of these parameters. Our estimates and the relevant previous estimates are found in the table below.

| | Ofgem IGT | | DPCR5 | CC re Bristol Water | | LE 2010 Estimates | |
|---|---|---|---|---|---|---|---|
| | Feb-02 | | Dec-09 | Jun-2010 | | Oct-2010 | |
| | Low | High | Final | Low | High | Low | High |
| **Cost of debt:** | | | | | | | |
| Risk free rate | 2.75 | 2.75 | 2 | 1 | 2 | 1.5 | 1.5 |
| Debt risk premium | 2 | 3 | 1.6 | 2 | 3 | 2.7 | 3.6 |
| Cost of debt | 4.75 | 5.75 | 3.6 | 3.9 | 3.9 | 4.2 | 5.1 |
| **Cost of equity:** | | | | | | | |
| Risk free rate | 2.75 | 2.75 | 2 | 1 | 2 | 1.5 | 1.5 |
| Equity risk premium for the market | 3.5 | 3.5 | 5.25 | 4 | 5 | 4.5 | 4.5 |
| Gearing | 37.5% | 37.5% | 65% | 60% | 60% | 37.5% | 37.5% |
| Equity Beta | 0.7 | 1 | 0.9 | 0.64 | 0.92 | 0.85 | 0.85 |
| Small company premium | 0 | 0.8 | 0 | 0 | 0 | 0.8 | 2.63 |
| Post tax cost of equity | 5.2 | 7.05 | 6.73 | 3.56 | 6.6 | | |
| Taxation adjustment (multiplier) | 1.43 | 1.43 | 1.39 | 1.39 | 1.39 | 1.39 | 1.39 |
| Pre-tax cost of equity | 7.4 | 10.1 | 9.3 | 4.9 | 9.2 | | |
| **Real pre-tax WACC** | **6.4** | **8.5** | **5.6** | **4.3** | **6** | **6.90** | **8.82** |
| **Vanilla WACC** | **5.03** | **6.56** | **4.69** | **3.76** | **4.98** | | |

***Source: LE and Ofgem***

### Debt risk premium

We present the details of our estimates in the section on the small company debt premium and the conclusions found there are the bases of our estimates in the table. We based the estimates on regulatory precedent, actual debt funding spreads for IGT companies, and our own empirical modelling of the debt premium. We estimate a range of 270 to 360 basis points.

### Cost of equity

The cost of equity is the risk free rate, plus any equity risk premia. The risk factors are a) the market, b) the small company premium and c) the growth/value premium. We have included a market premium along the lines of the Sharpe-Lintner CAPM (standard CAPM) and a small company premium using a UK-data-based Fama and French 3-factor model.

*Market beta and market equity risk premium*

The market beta estimates we use come from Ofgem's previous 2002 results; we use the mid-point of Ofgem's range (0.85). (We note that the market betas in our own 3-factor model regressions had coefficients close to 1.0, but it is our judgment that regardless of methodology, the correct value should most likely be close to 1.0. We note that the previous Ofgem IGT decision used a value of 1.0.)

Further, the cyclical risk of the IGT sector relative to the market has likely increased since 2002 (indicating a possibly higher beta). The cyclicality of the sector is important as it indicates possible non-diversifiable risk (and not captured in standard utility proxy betas). The cyclicality is due to the strong positive correlation between new connections and new construction. Also, unlike standard regulated utilities, the AIGT companies do not have the ability to pass-on variations in opex and other on-going spend, either via the formula (e.g., indexed to cost-profiles, RPI, etc.), or upon review every five years and resetting the base price/allowed cost.

Nonetheless we've used for our beta the midpoint of the values 0.7 to 1.0 from Ofgem's 2002 IGT decision, 0.85. We would highlight that our beta is lower than the previous beta used by Ofgem in 2002, but rather than adjusting beta upwards by choosing the top of the range, we estimate a small company premium directly. We note that the standard CAPM generally works well for standard utilities, but for small companies, it is our judgement that the standard CAPM does not work well.

The market risk premium comes from the Bristol water appeal, mid-point of the range (4 to 5) of the CC decision, 4.5%.

*Small company premium*

As we have taken mid-point values from the previous regulatory decisions, we have focused on the small company premiums for equity and debt.

Our estimates of the small company equity premium are based on:

1) The previous Ofgem IGT decision of 2002, and the fact that the IGT market has become more risky, and that financing for small firms has become more challenging.

2) Long-term estimates of the small company premium based on company returns in the UK and elsewhere.

3) Academic evidence.

4) Our own regression models of the cost of equity and the size premium.

5) Smallness may proxy for illiquidity, growth/value, market inefficiency, and variety of factors. The practical notion and professional judgment is that small firms are more risky, and that this additional risk is not diversifiable, and should therefore earn a premium rate of return. We have not double counted the possible additional premia that may exist.

We estimate the small company equity premium to be 80 bps to 263bps.

**Pre-tax WACC**

In response to Ofgem's consultation letter, we have estimated the cost of capital for the IGT sector legacy assets.

We have focussed on the pre-tax cost of capital. Our estimates of the WACC are in the range from 6.90% to 8.82%.

The IGT sector is fundamentally different from other sectors and/or companies that Ofgem regulates because a) it is competitive, b) it is very small,c) the price control for legacy assets is purely based on the cost of capital c) does not have pass-through on non-controllable costs d) is more cyclical, and e) has certain regulatory constraints and differences which are more risky than GDNs (e.g. metering).

Given the above, and also given recent decisions of Ofgem, other UK regulators (e.g., Ofwat) and findings of the Competition Commission (CC) from the Bristol Water appeal, we have only focused on estimating a small company premium for the cost of debt and the cost of equity. For all of the other WACC parameters we have relied on previous estimates. For the gearing and beta assumptions, we relied on the previous Ofgem IGT sector WACC; for the risk free rate, and the market risk premium (which are more time-sensitive), we relied on the recent CC findings in the Bristol Water Ofwat appeal.

For the small company debt premium, we provide LE's own estimates. The estimates are based on a) the actual data on recent borrowings in the sector b) previous regulatory findings—and the fact that the financial crisis has most likely raised the cost of debt relative to the previous 2002 Ofgem Decision, c) our own modelling of debt spreads using two different approaches (regression using debt spreads and UK bonds, and using CDS spreads), and d) our own professional judgment. The sum of this evidence suggested a debt spread in the range of 270 to 360 basis points over the risk free rate.

The debt spread we use includes a small company fees/costs adder of 10bps on the cost of debt, as there are fixed elements of fees for raising debt. We would note that realised total fees on debt for some IGT companies are likely to be higher. Evidence from the IGT members confirms the experience of smaller companies/smaller loan amounts incurring additional arrangement and commitment fees of circa 26bps. While we have seen evidence from the various AIGT companies that fees could be significantly higher than 10bps, because of the confidential nature of these, and also because of the difficulty in converting fees, such as non-utilisation fees[1], into a basis points/percentage point adder, we have not included these.[2]

For the cost of equity, we also have provided our own estimates. The estimates are based on a) existing international, professional, and academic research on the small company premium, b) previous regulatory findings, c) our own modelling of the cost of equity small company premium using two different approaches (regressions using the Fama and French 3-factor model; regressions on UK data using the Fama and French methodology. We also present data on UK company betas and company size classes), and d) our own professional judgment.

[1] It would be very difficult to estimate, what percentage of the loan facility is expected to be drawn down over the period; this would then have to be spread over the period as an expected value, and converted to a percentage by the total (uncertain) drawdown, for example.

[2] We have advised the AIGT companies to consider liaising with Ofgem directly on these fees and evidence thereof.

Overall, we suggest that the upper end of the range presented is the correct one for Ofgem to choose. This is because a) the IGT firms face risks that are higher than normal regulated firms b) the overall approach to regulation of the legacy assets of IGT firms is of a reasonable profits test based on WACC, rather than a standard full price control, and c) the costs/prices of the IGT sector have already satisfied an additional hurdle rate given that these are companies that are fully competitive when bidding for new sites d) Ofgem previously chose the upper end of the range, e) the financial crisis means that risks relative to 2002 levels are now much higher, and f) we have been careful not to overestimate the premia involved and the upper end of our small company premia are close to our point estimates for these parameter.

# 1 Introduction and background

The Association of Independent Gas Transporters (AIGT) has asked London Economics to prepare a report on the topic for potential inputs to the Ofgem consultation on cost of capital methodology for IGTs.

Ofgem is now reviewing the cost of capital methodology in the reasonable profits test for independent gas transporters so as to ensure charges subject to the reasonable profits test continue to represent good value for consumers. Ofgem recognises that the reasonable profits test should take into account the risks associated with the business of the IGT sector, as well as financing costs.

## 1.1 Background

### 1.1.1 AIGT companies overview

The IGT sector in the UK represents a truly innovative approach to regulation and introducing competition into a sector that for most countries would represent a 100% regulated sector.

The IGT companies in the UK with legacy assets are small by UK utilities standards. Book values of tangible assets for the companies on a consolidated basis range from about £145m to £175m, whereas larger UK utilities have assets and market caps over £1bn.

In other terms, the book value of total assets net of liabilities for these companies is very small, and ranges from several million pounds negative to between £20m and £40m positive.

### 1.1.2 Ofgem's Consultation

Ofgem regulates the rate of return on certain (legacy) assets of the IGT sector. The rate of return is based on a WACC approach, with the WACC set in a 2002 Ofgem Decision. Ofgem is now considering the updated WACC as Ofgem had agreed to review the WACC periodically.

Ofgem has stated to stakeholders to the consultation that they are particularly interested in the following.

- The particular business risks associated with this type of network operation;
- Factors affecting the ability of IGTs to raise debt or equity finance at competitive rates; especially with reference to any characteristics of legacy network sites;
- Factors affecting IGTs' gearing levels;
- Equity Beta;
- Financing issues affecting IGTs as relatively small businesses; and
- The risks associated with bad debts or loss of revenues on legacy sites.

We consider each of these in turn and then, as will be argued, where warranted additional details and analysis are presented in subsequent sections.

***Particular business risks***

Ofgem has requested consideration of the particular business risks of the competitive gas suppliers with legacy assets.

We note first that the risks of these companies are quite different than those of more standard regulated companies. The companies in the IGT sector are small, competitive and in a growth/new growth phase; i.e., the competitive companies must continually win new customers, as small companies have different access to finance and new growth companies have less track record, more need for up-front cash flows. Also unlike standard regulated utilities, the AIGT companies do not have the ability to pass-on variations in opex and other on-going spend, either via the formula (e.g., indexed to cost-profiles, RPI, etc.), or upon review every five years and resetting the base price/allowed cost. Additional expense items that, according to AIGT, have increased that might typically be considered non-controllable (and thus passed on via the next price control review) for standard regulated firms include items such as local rates, taxes, fees (regulatory), etc.

An additional factor is that the legacy assets in question are sunk costs, in that assets are in the ground locally. The 'sunkness' of the assets comes into play particularly with respect to metering and the eventual replacement of meters with smart meters. It is not clear who will own the future smart meters and what will happen to (standard) meters which are effectively sunk and stranded. This stranded meters issue also interacts with the fact that IGTs have a licence condition that makes them the meter provider of last resort—so an IGT might be forced to install a meter they'd not otherwise have considered commercially viable.

The risks involved with meters are of particular concern to the IGT companies, because a) there are economies of scale and scope[3] in their opex, b) their business plans/financing are dependent on this and c) they could have stranded (old) meter assets and not own new smart meters. We note that the metering issue is fundamentally different for IGTs as compared to GDNs and suppliers, and National Grid, where National Grid took over ownership of the meters. National Grid (Transco) has already had compensation for the glide-path in metering, and was given an allowance for this in their 2002 price control. Also, the NG meter asset portfolio is considerably older than IGT's assets (and so less vulnerable to stranding).

A particularly challenging point in terms of the regulatory process is the acceptability of the fundamental logic of the basic CAPM cost of capital approach to allowed costs—where only non-diversifiable market risk is allowed a premium—for small unlisted firms. In general, for the cost of equity of such firms, it is an issue as to whether market idiosyncratic risk—that is to say, particular firm risk, should be given a risk premium or whether there should be additional risk factors.

A first response might be that for such small firms, the particular or idiosyncratic risks are not diversifiable, because in general, such small firms are not traded or not listed, and in general, their debt is not traded or listed either (typically bank debt and equity capital is their means of financing). Thus, these firms should deserve an illiquidity premium.

Further, the "plain" or standard CAPM simply takes the only risk factor to be market risk, and so (possibly) assumes away the possibility of a small company premium. While it is possible that the beta risk of a standard CAPM captures well the non-diversifiable risk of small companies, the available empirical evidence suggests otherwise.

In general, then, while it might seem reasonable to wholly dispense with a CAPM-WACC framework for such small companies, we in general avoid this, because:

[3] While LE has not conducted a detailed study into the estimation of the economies of scale and scope, we are informed of this by the AIGT companies. It is our judgement that this seems reasonable as small companies would typically have high levels of scale and scope economies (whereas larger companies typically exhaust these at some size and shape).

a) The previous work done by Ofgem in 2002 set allowed revenues based on a CAPM-WACC foundation;
b) Most regulators use CAPM-WACC, including Ofgem; and
c) CAPM-WACC allows verifiable company evidence and international evidence and is in-line with best practice in terms of corporate finance practice and academic research.

For these reasons, we focus much of our attention on the multi-factor CAPM approaches of academics such as Fama and French (for the cost of equity), and models of risk and default risk spreads for the cost of debt, but staying within the WACC framework.

Nonetheless, we still would re-emphasize that the particular business risks of small and competitive gas (and some have electric, dual fuel, etc.) companies would be in general more risky than for standard utilities. Perhaps the largest source of this would be that most of their connections/new connections are coming from new estates/housing/buildings. The IGT sector's main source of growth is from connecting new customers in new estates, apartment blocks, business parks and complexes, and similar new-build construction. These are related to the construction cycle which is well known to be highly cyclical. Cyclicality and the general business cycle is also believed to be not just a risk, but often a (at least partially) non-diversifiable risk, as usually most businesses including the stock market are impacted by the cycle.

Conversely, standard utilities are in general known to be somewhat non-cyclical—people vary their energy use very little with the economic cycle (some other factors such as bad debts are different). Further, the IGT sector companies will not in general have a standard price review which will "allow" opex or capex costs to be recovered. Thus betas and other risk factors based on standard utilities might under-estimate the risk for the IGT sector.

***Factors affecting companies' ability to raise debt***

In general, the companies with legacy sites face the following hurdles when raising debt:

1) Current economic conditions;
2) Company size;
3) Size of financing required; and
4) Regulatory uncertainty.

We take the following topics up in a more detailed chapter that follows.

In general, small companies find it harder to raise debt capital than larger companies. In general as well, companies will have minimum scales for particular financings in order to tap bond markets, more liquid markets. We provide a more detailed discussion of the impact of company size on debt spread and on debt availability.

In the long run, bankers and debt holders will want to see a high degree of certainty for the revenue streams of the companies. The regulatory process and its results can thus either help or hinder the sector. We would note that the 2004 Ofgem Decision included the review of the cost of capital every five years. Further, while the risks of not having a regulatory review, with allowed/regulated levels of cost for opex is naturally not part of the process, this element is similar to normal, e.g., non-regulated business, and so it might merely be argued that this could impact the perceived level of credit risk, rather than the overall ability to raise finance.

Another factor for the IGT sector is their reliance on short-term to medium term financing and thus facing refinancing risk regularly.

***Factors affecting the AIGT companies gearing levels***

Because the IGT sector companies are so small, effectively, their gearing is related to their parent companies' ability to raise debt and financing. Thus, it appears that tax treatments and intra-company rationales are the main factors driving IGT companies gearing. We would note, however, that for the regulatory WACC the gearing is for a) the sector and b) not supposed to be prescriptive. Further, the gearing of the companies is somewhat difficult to calculate as a) there are a number of companies in the sector b) there are variable structures of the companies, but the companies are ring-fenced c) the companies have been growing, d) the companies are not listed so, book values of equity and assets are all that are available.

As there are a number of factors impacting the gearing, and as the regulatory approach to gearing has in general been a target gearing that would be achievable by the typical company while maintaining a good credit rating, we have not investigated the gearing of the companies in detail.

***Equity beta***

The AIGT companies do not have listed shares, and so equity betas must be found on a proxy basis. The equity beta refers to the Capital Asset Pricing Model (CAPM) measure of undiversifiable market risk, and so is a proportional measure of the premium for a company over/under the standard market equity risk premium.

The equity beta can also be thought of as the beta (regression coefficient) on the equity market excess returns—this is true for the standard CAPM and for multi-factor asset pricing models.

The equity beta for small companies is generally not observable/estimable because such companies tend not to be listed.

Small companies earn an equity premium relative to other companies. We present a variety of detailed evidence on this subject but we believe that this is unequivocal. Thus, the CAPM equity beta does not capture all the relevant risk premia for the companies at hand. We present more detailed evidence on this in the section on the cost of equity.

We note that an alternative approach to the using a small company premium could have been to use a slightly larger equity beta within a standard CAPM WACC (e.g., just one factor "the market"). We, however, have not chosen to do this, as the evidence on the small company premium we find more compelling. Part of the rationale for this is that the market betas, from proxy utility companies, are likely to underestimate the true market beta for IGT companies. This is because IGT companies tend to be more risky, but also more cyclical. We have used a conservative equity market beta in our estimates.

***Financing issues affecting IGTs as relatively small businesses***

The IGT sector in general will face issues affecting their business and financing. None of the IGT sector companies currently has traded shares, nor would be likely to raise significant funds by tapping the traded equities markets in the near future. Further, the IGT sector companies with legacy assets are also most likely too small to access bond markets in the next few years, and thus achieve the lower costs of debt associated with those more liquid markets. Bank finance and various forms of syndicated and artesian debt are all that these companies are likely to be able to achieve in the short to medium term.

In the context of the financial crisis, the access to finance for small companies has in general diminished.

An additional and important issue for the IGT sector is the nature and type of loans that they have currently and that are available. A number of the IGT firms have bullet loans, rather than more blended types of debt finance, with the result that a majority or even all of a given company's debt must roll over at a single time. This is considerably more risky to the company. It is also noteworthy that this is unlike most utilities. The lack of track record of these relatively new sector firms is one of the main reasons bullet loan type debt finance has been used.

Fees on debt are also an issue for the IGT sector, and some fixed portions of arrangement fees necessarily means additional unit costs of debt for small loan amounts.

Finally, the availability of information, collateral, and other factors such as the track record/age of these companies indicates the possibility of credit constraints, and or higher borrowing costs.

***The risks associated with bad debts or loss of revenues on legacy sites***

The risks associated with bad debt have increased with the financial crisis. The main risks the IGT sector faces relate to possible defaults by shippers[4] or possible defaults by suppliers, other companies, or companies such as construction companies who may have done damage to pipelines and equipment.

Relative to the standard GDN, IGT sector risks with respect to bad debt might be higher. A notable point is that in the case of insolvency for the wider regulated sector, Ofgem has given an undertaking to GDNs regulated under the normal price control that they will be able to recover bad debt from defaulting shippers and suppliers through subsequent price controls (provided the GDN has engaged best practice billing processes). No such arrangement is in place for the IGT sector, either under the relative price control (to which companies' assets will eventually migrate) or the legacy site WACC-based reasonable profits regulation.

### 1.1.3 Are the IGT companies with legacy assets "small firms"?

An important initial question in regards to a small company premium is how a small company is defined and whether the IGT companies in general fall into this category. We argue that these are small companies by almost any definition with regards to the finance literature on small company premia.

Perhaps the most widely cited group of articles and research on small company premia, the Fama and French (F&F) three factor models (FFTM), define a small company as between the 20th percentile and 50$^{th}$ percentile of the market cap of stocks listed on the NYSE (their sample includes more indices, AMEX, Nasdaq, etc., but they define size based on NYSE percentiles). Further, in an additional article "Dissecting Anomalies", (Fama and French, 2007)[5] define "micro-cap" stocks as stocks with market cap less than the 20th percentile of the NYSE.

Given the F&F definition, the average book value of the tangible assets of legacy IGT firms is between circa £145-£175m. However, the profits of most of these companies are negative or very small, and further, total assets less total liabilities for these companies tended to be in the range of circa £20 to £40m (for those with non-negative values).

[4] The shippers effectively use the IGT company's network to deliver their gas that has been purchased. There would be a billing cycle for this, so if a shipper were to suddenly go out of business, the IGT company could be left with a bad debt.

[5] Fama, E, and K., French, "Dissecting Anomalies", CRSP Working paper #610.

Other definitions of size of course can be considered. For most international and investment fund managers, the Russell 2000 (R2000) small cap definition is what is used. Russell 2000 small cap defines a small cap as the smallest 2000 stocks in the Russell 3000 index. While naturally then the actual market cap (and thus the market cap of the smallest 2000) will change with share prices, the weighted average market cap in the R2000 was about $1bn, or £620m, or circa 4 to 5 times the size of the typical IGT legacy firm. We would note that the upper end of the range would form the limits as to what is a 'small cap' and what is not, rather than the average. But even under this definition these are small companies.

Another definition, FTSE defines a global small cap index and also a UK small cap index and defines this as the smallest 10% of their Global and UK equity indices. For the London Stock Exchange, there are two markets. For the Main Market, the following distribution is found. From the figure, it can be ascertained that roughly any company with less than £500m will not be in the top 5% of companies by market capitalisation listed on the Main Market. We note, however, that there are a number of markets in the UK, and even on the LSE, so as the Main Market tends to have the larger firms, if anything restricting the analysis to the Main Market is not likely to underestimate the portion of the market that contains small firms.

**Figure 1: Size distribution market cap, LSE Main Market**

| Market value range (£m) | Main Market - UK Listed: No. of companies | % | Equity market value (£m) | % |
|---|---|---|---|---|
| Over 2,000 | 117 | 11.5 | 1,573,769.8 | 86.3 |
| 1,000 - 2,000 | 69 | 6.8 | 99,410.7 | 5.4 |
| 500 - 1,000 | 91 | 9.0 | 65,891.1 | 3.6 |
| 250 - 500 | 120 | 11.8 | 42,926.9 | 2.4 |
| 100 - 250 | 176 | 17.3 | 28,959.0 | 1.6 |
| 50 - 100 | 104 | 10.2 | 7,382.7 | 0.4 |
| 25 - 50 | 100 | 9.9 | 3,795.7 | 0.2 |
| 10 - 25 | 99 | 9.8 | 1,708.3 | 0.1 |
| 5 - 10 | 53 | 5.2 | 406.6 | 0.0 |
| 2 - 5 | 26 | 2.6 | 93.1 | 0.0 |
| 0 - 2 | 28 | 2.8 | 27.1 | 0.0 |

***Source: LSE website***

Studies and size measures focussing on the cost of debt can define size in different ways. Damadoran (2001)[6] assumes credit rating and interest coverage ratios as sufficient to give a direct mapping for credit spreads. He maps different credit ratings for the same financial ratios (interest coverage) for hypothetical same firms based on size, and splits size based on a market cap of $5 billion (circa £3billion). For a 50/50 gearing ratio this would give a total firm value of circa £6billion.

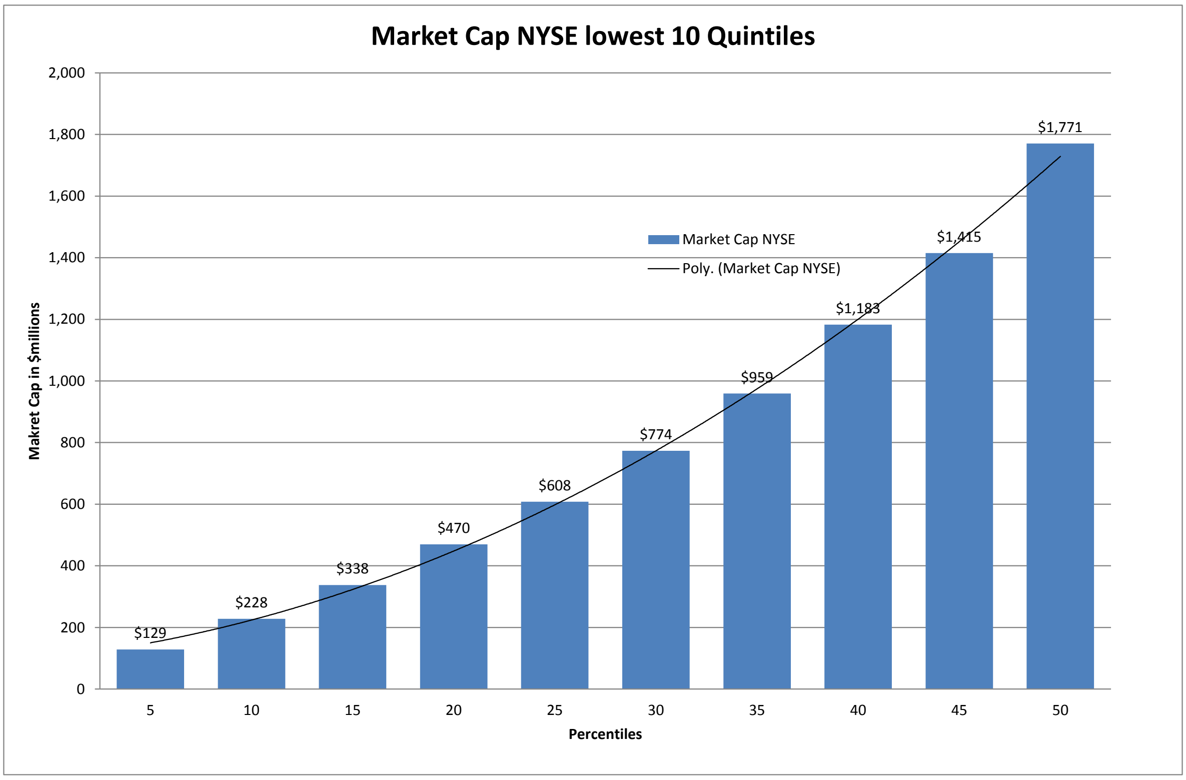
For our own study of size and debt spread, we simply defined size as the smallest 10% of firms (by market cap) in the sample (and our prediction was based on the median of this group). This would

[6] Damadoran, A, Investment Valuation, New York: Wiley. 2001.

give a market cap of approximately £400m, that is, firms with market cap below £400m were classed as small companies, and the prediction was based on the average of this group, which was about £131m.

Additional evidence on the size of firms can be had using NYSE data. The figures below give the quintiles for the NYSE market caps at the end of 2009. The first figure gives the lower 10 quintiles.

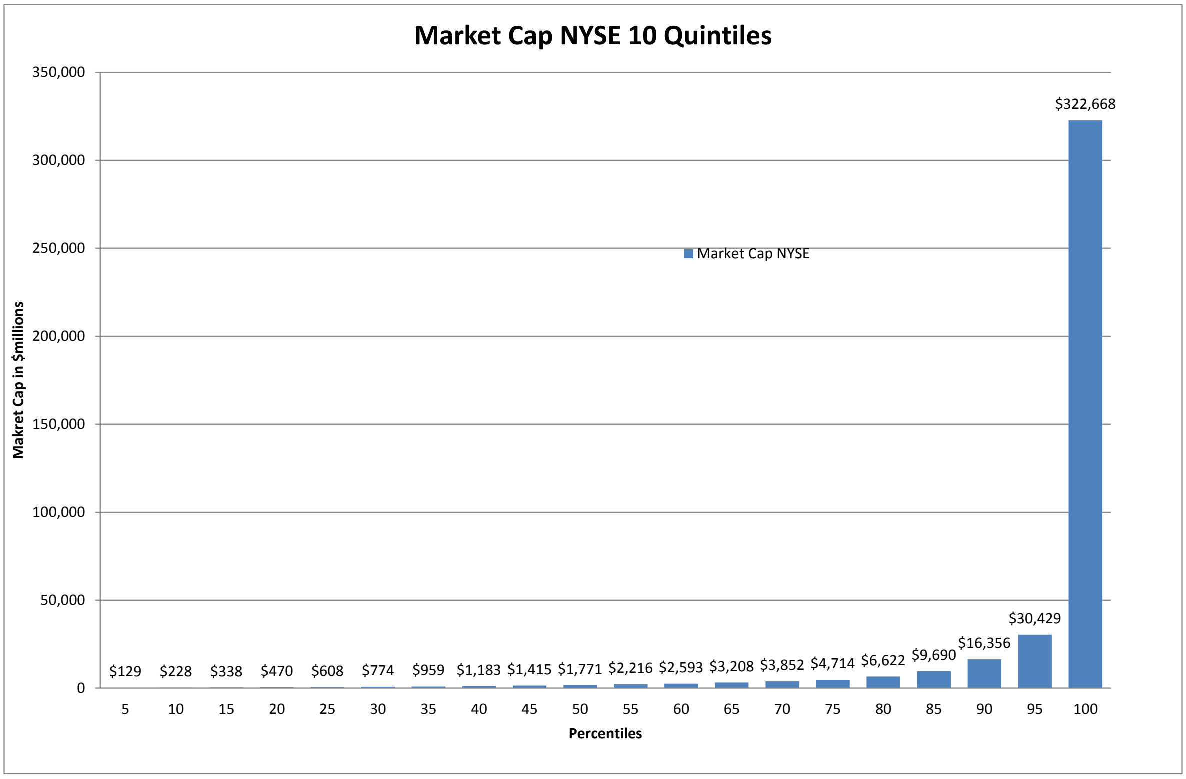
**Figure 2: NYSE smallest 50% of companies size distribution**




Note:

***Source: Fama and French data library online***

**Figure 3: Size distribution NYSE**




Note:
***Source: Fama and French data library***

Thus, based on the above size definitions, firms below the 20th percentile would have a market cap of less than $470m or about £291m—these would be considered micro caps. Firms with between about £275m and £1100m would be considered 'small caps'.

A final note on the IGT companies' size is to recall that the subsidiary companies are effectively ring-fenced from the parents, and so the parent companies would not be the measure by which to gauge company size, it should be noted that parent companies will regularly require high rates of return from small subsidiaries when deciding which parts of any business get capital.

# 2 Regulatory Decisions on Small Company Premia

As stated in the introduction, we do not review every aspect of the recent regulatory experience but focus on the small company premia. We note the recent appeal of the Ofwat decision and the CC preliminary findings taken as cited by Ofgem in their IGT consultation letter (and the subsequent final decision). We therefore focus *only* on the small company premium aspect of the cost of debt also any tertiary aspects such as fees. We note that the level of the premium allowed might be something that changes empirically with market conditions, but that the acceptability of the premium is the main usefulness of this evidence.

## 2.1 Ofgem

Ofgem has previously allowed a small company equity premium in 2002 for the WACC calculation for the IGT sector. This value was set at 0.8%.

## 2.2 Competition Commission (CC)

### 2.2.1 Ofwat/Bristol Water appeal

The Competition Commission has set out its views in various appeals of regulators' decisions in the UK. Among the most recent, the appeal of the PR09 Ofwat decision, the CC set out a range of the cost of capital. In the table from the Ofgem IGT consultation letter, the table appears to indicate that the CC did not allow a small company premium in the Bristol Water appeal, nor did Ofwat allow a small company premium. However, the CC appeal document clearly recognised that Ofwat set different costs of capital in three size bands based on company size (Regulatory Capital Assets). According to the CC document, "The cost of capital may vary according to the size of company. Ofwat's approach involves setting the same cost of capital for each company in three size categories. Ofwat's approach is therefore based on comparative rather than individual company data. Ofwat has three size categories:

(a) large, including all the WaSCs;

(b) larger WoCs (South East Water, Veolia Central); and

(c) other smaller WoCs (including Bristol Water).

Ofwat sets a higher cost of capital for smaller companies. Thus Ofwat sets a higher cost of capital for Bristol Water than the two larger WoCs and the WaSCs. Ofwat's approach to cost of capital differs from its approach to other elements of required revenue (such as opex and capex) in that it allows higher costs for smaller companies."[7] Overall, the CC accepted that variability in opex versus capex was the main rationale for this, rather than a size premium *per se*.

[7] http://www.competition-commission.org.uk/inquiries/ref2010/bristol/pdf/appendices_and_glossary_merged.pdf, para 22.

### 2.2.2 Domestic bulk LPG

An interesting comparator to the IGT sector is the UK domestic bulk liquefied petroleum gas (LPG) sector (generally propane gas, with mixtures of butane and other gases). The CC reviewed the WACC and the cost of capital of the LPG sector as part of their inquiry in 2005-2006. The main thrust of the inquiry was into practices which discouraged switching suppliers, such as up front charges for tanks (the CC required remedies to address these), but the CC assessed the WACC of the LPG sector as well. Again, an interesting comparative aspect between the IGT and LPG sectors is there is a wide range of suppliers in LPG, including small companies, and the LPG supply sector is competitive (and also a substitute for gas delivered by pipe). There may be some fixed and sunk assets, such as tanks, although the asset base is likely to be smaller in LPG as there is not fixed network.

In its findings on WACC, the CC allowed a 0.5% addition to the cost of debt for small companies in the LPG sector.

## 2.3 Other regulators

### 2.3.1 Ofwat

In Ofwat's PR04 final determination, water only companies were allowed both an equity and debt small company premium. Ofwat recognized in principle three types of small company premium:

"The 'small company effect' can be analysed in terms of three broad components:

- An equity return premium to compensate for higher trading costs;
- An interest rate premium on the cost of debt finance for water only companies relative to water and sewerage companies;
- Premiums on the costs of raising capital (for both debt and equity)."[8]

Ofwat, in PR04 guidance, also noted that they had allowed a 0.75% premium in their 1999 price review. They then noted that the small company premium could be a function of illiquid trading, "In contrast a premium on the cost of equity remains valid, principally to address the fact that there is a less liquid market in the water only companies' shares."[9]

The total for the premium was 0.3 per cent to 0.9 per cent on a post-tax basis, with the premium depending on the size of the company which could fall under any of four different size bands, using regulatory capital asset values as the measure of size. Ofwat used the following size bands (in 2004 £s).

[8] http://www.ofwat.gov.uk/pricereview/ltr_md190_gudfinalbusplans

[9] Ofwat, op, cit.

**Figure 4: Small company size classes, Ofwat PR04**

| Regulatory Capital Value |
|---|
| <£70m |
| £70m to £140m |
| £140m to £280m |
| £280m to £700m |

Note: Data available on the Ofwat website.
***Source: Ofwat***

**Figure 5: Ofwat size premia by class PR04**

| RCV | Companies | Premiums: Total – Gross of tax shield (%) | Premiums: Total – Post-tax (%) | Premiums: Equity – Post-tax (%) | Premiums: Debt – Pre-tax (%) | Premiums: Debt – Post-tax (%) |
|---|---|---|---|---|---|---|
| < £70m | Cambridge, Dee Valley, Folkestone & Dover, Tendring Hundred | 0.9 | 0.9 | 1.5 | 0.5 | 0.4 |
| £70m - £140m | Bournemouth & W Hampshire, Portsmouth, Sutton & East Surrey | 0.8 | 0.7 | 1.3 | 0.4 | 0.3 |
| £140m - £280m | Bristol, Mid Kent, South Staffordshire | 0.7 | 0.6 | 1.2 | 0.3 | 0.2 |
| £280m - £700m | South East and Three Valleys | 0.3 | 0.3 | 0.5 | 0.1 | 0.1 |

*Source: Ofwat*

Note: From the Ofwat website
***Source: Ofwat***

Ofwat allowed a cost of debt and cost of equity small company adjustment in its 2009 draft determination. It allowed a cost of debt uplift of 0.4% (40bps) for water only companies but in their final determination they did not allow a small company premium *per se,* but allowed different costs of capital by three different size bands (see CC subsection above).

### 2.3.2 NIAUR

The Northern Ireland Utilities regulator, NIAUR, system operator of Northern Ireland, SONI, price control determination of 2008, included a small company premium. NIAUR published its decision paper on SONI price controls covering the regulatory period 2007-2010. NIAUR included in the cost of capital for SONI a small company premium of 0.265 per cent.

### 2.3.3 Oftel

The Office of Telecommunications Regulation in the UK has on a number of occasions considered a small company premium. Perhaps the interesting case is that Oftel allowed a small company premium in their assessment of the cost of capital for mobile in 2002. These companies are similar to IGT companies in that they are both small, new/growth, and competitive. Oftel allowed a 1.35% added premium to the cost of equity on a post-tax basis (at the current tax wedge, this works out to circa 1.88% pre-tax premium—(i.e., using 1.39 as the tax wedge multiplier).

### 2.3.4 CREG (Belgian Energy Regulator)

The CREG is the Commission de Regulation d'Electricité et du Gaz, the energy regulator in Belgium. The CREG has a similar regulatory programme to Ofgem in that they are introducing competition to some parts, allowing regulated tariffs to other parts of the energy network. In 2004, the CREG approved the tariffs for gas distribution connection in Belgium, and allowed an "illiquidity" multiplier, which amounted to a premium of between 0.7 and 1.0%.[10]

### 2.3.5 Fees and other costs of capital

The Civil Aviation Authority (CAA) in its 2007 Determination on Gatwick included the cost of fees and arrangement costs in their cost of debt.

The CC, in their review of the Stansted 2009 Decision by the CAA, allowed 0.1% on the cost of debt for fees.

## 2.4 Conclusions to this section

In general, regulators have not tended to allow small company premiums to the cost of capital within the CAPM framework. We suspect a large portion of the reasons for this is that regulated firms are normally not that small, or at least borderline small. For larger companies, the standard CAPM framework tends to work reasonably well. In addition, regulators have tended to focus on the overall cost of equity and the overall cost of debt (rather than splitting out a small company premium), and have allowed size variations based on opex and capex profiles, arrangement fees, and other factors, rather than size alone.

In the UK, Ofgem, Ofwat, Oftel and NIAUR have allowed small company premia.

Previously Ofgem allowed a 0.8% premium to the cost of equity for the IGT sector. Regulators may have allowed small company premia implicitly in some cases by adjusting upwards standard WACC parameters. Ofgem did not allow an explicit uplift for the cost of debt, but we note that the allowed debt premium from the 2002 IGT cost of capital was between 200 and 300 bps, which is higher than the Ofwat 160bps most recently reviewed/allowed by Ofwat or the CC.

[10] See CREG (2004) available at : http://www.creg.info/pdf/Divers/TG2004FR.pdf

# 3 Cost of Equity

This section reviews evidence on the cost of equity and a small company premium.

## 3.1 Existing research on small company equity premia

### 3.1.1 CAPM, big companies and the size premium

For large, liquidly traded listed companies, or portfolios of companies, the standard single factor CAPM is likely to capture risk reasonably well. CAPM has been the tool of choice for regulators around the world when setting the cost of capital for large regulated firms. However, the CAPM does not always perform as well as other models of asset returns, especially when considering small and/or growth companies. This has led to a large body of financial economics research. Thus while we accept that a standard CAPM is likely the most reasonable approach in most standard regulatory cases (and when larger companies are involved), for the IGT sector we believe a standard CAPM may not be the most appropriate model of the cost of equity capital.

The existence of the size premium is evident from basic analysis of UK stock market returns data, as the table below demonstrates.

**Table 1: Summary statistics monthly returns UK stocks 1980-2008**

| Five portfolios sorted into quintiles based on size (market cap) Equally weighted portfolios | | | | | |
|---|---|---|---|---|---|
| Variable | S1 | S2 | S3 | S4 | S5 |
| Mean | 2.08% | 1.22% | 1.11% | 1.02% | 1.03% |
| Std Dev | 5.32% | 5.19% | 5.32% | 5.45% | 5.26% |
| z-stat | 7.19 | 4.31 | 3.84 | 3.46 | 3.61 |
| Value-weighted portfolios | | | | | |
| Variable | Sv1 | Sv2 | Sv3 | Sv4 | Sv5 |
| Mean | 1.82% | 1.20% | 1.10% | 1.01% | 1.01% |
| Std Dev | 5.35% | 5.17% | 5.31% | 5.46% | 4.65% |
| z-stat | 6.27 | 4.28 | 3.80 | 3.41 | 3.98 |

From the table, it is clear that the average return on a portfolio increases as size decreases.

A t-test of the difference between two sample means indicates that the smaller quintiles' returns are significantly above the largest quintiles returns for both the value-weighted and equally weighted portfolios.

We consider additional evidence on the UK small company premium later, but first, we review the literature on the small company premium more generally.

### 3.1.2 Fama and French and the size premium

Financial economists have studied the CAPM and have shown that the explanatory power of CAPM does not always perform well (or as well as it could). One of the most well-known and accepted anomalies is the "size effect" first detailed by Banz (1981)[11], who found that the average returns of smaller US companies appeared high relative to the returns of larger firms, even when accounting for market risk as implied by the CAPM framework. This work was further investigated by Fama and French (1993)[12], who found that two variables, size and book-to-market value, capture most of the variation in stock returns not captured by the standard CAPM framework. Fama and French proposed the Fama French three-factor model ("FFTM") that attempts to adapt the conventional CAPM by adding additional explanatory variables for size and book-to-market value.

Fama and French (2006) examined the way in which value premiums varied with firm size. Constructing Value minus Growth (VMG) variables from six portfolios based on market capitalization (Low or High) and book to market equity (Growth, Neutral or Value using 30th and 70th percentile breakpoints), the Fama-French (2002) three factor model was used to test whether the value premium in average returns is a particular feature of small cap stocks. Estimation was based on data on NYSE, AMEX and NASDAQ firms with positive book value between 1926 and 2004. The table below shows summary results of the modelling process.

Whether the size premium could be found internationally was also considered by Fama and French and others, according to Fama and French (1997)[13], "Researchers have identified several patterns in the cross-section of international stock returns. Heston, Rouwenhorst, and Wessels (1995)[14] find that equal-weight portfolios of stocks tend to have higher average returns than value-weight portfolios in 12 European markets. They conclude that there is an international size effect."

[11] Banz, Rolf W. (1981) "The relationship between return and market value of common stocks", March, Journal of Financial Economics.

[12] Fama, E., French, K., (1993) "Common risk factors in the returns on stocks and bonds", Journal of Financial Economics.

[13] Fama, E, and French,K., (1997), "Value versus Growth: The international Evidence" Tuck Business School working paper.

[14] Heston, Steven L., K. Geert Rouwenhorst, and Roberto E. Wessels, 1995, The structure of international stock returns and the integration of capital markets, Journal of Empirical Finance 2, 173- 197.

**Figure 3.1: Summary Statistics for Monthly Returns on Size and Value Factors in Factor Protfolios, 1926-2004**

| | SMB | VMG | VMGS | VMGB | VMGS-B |
|---|---|---|---|---|---|
| **July 1926 - December 2004** | | | | | |
| Mean | 0.23 | 0.4 | 0.48 | 0.31 | 0.17 |
| SD | 3.36 | 3.58 | 3.63 | 4.25 | 3.33 |
| T-Statistic | 2.06 | 3.43 | 4.08 | 2.23 | 1.6 |
| **July 1926 to June 1963** | | | | | |
| Mean | 0.2 | 0.35 | 0.35 | 0.36 | −0.01 |
| SD | 3.48 | 4.17 | 3.89 | 5.23 | 3.86 |
| T-Statistic | 1.23 | 1.78 | 1.89 | 1.46 | −0.08 |
| **July 1963 to December 2004** | | | | | |
| Mean | 0.24 | 0.44 | 0.6 | 0.26 | 0.34 |
| SD | 3.26 | 2.96 | 3.39 | 3.12 | 2.76 |
| T-Statistic | 1.68 | 3.34 | 3.97 | 1.87 | 2.76 |

*Source Fama and French (2006)*

From the SMB (Small Cap minus Big Cap) column, there is clear evidence of a size premium, across the entire sample and in the two sub-samples. The average small cap premium is 0.23% per month from 1926 to 2004, with the 1963-2004 sub-sample showing the highest per-month small cap premium of 0.24%. Accounting for the value premium, returns to small cap stocks are again higher, with a per-month premium of 0.48 (0.17% higher than the large-cap premium of 0.31%). Thus, both the sub-samples and the full sample confirm the existence of a small cap premium in the expected returns, regardless of whether one accounts for the value premium.

In a more recent paper, Fama and French (2007) 'dissect anomalies' in the CAPM and stock returns and see if they can uncover additional data or "factors" that explain returns in portfolios. Among the evidence they present is the possibility that there is in fact a "micro-cap" premium. They define a "micro" company as a company whose market cap is less than the 20th percentile of the NYSE market caps. The data are found in the table below. From the table, it is clear that there is a micro-cap premium in returns (the average micro-cap monthly return is 1.29), and that the risk (as measured by the standard deviation in returns) is highest (standard deviation is 6.99).

**Table 2: Summary stats small company premium**

| Average | Monthly | Values, | July | 1963 | – | December | 2005 |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | Firms | % of total market cap | Ave | Std Dev | Ave | Std Dev | |
| Market | 3060 | 100 | 0.94 | 4.44 | 1.36 | 6.14 | 15.14 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Micro | 1831 | 3.07 | 1.29 | 6.84 | 1.56 | 6.99 | 17.51 |
| Small | 603 | 6.45 | 1.22 | 6.03 | 1.21 | 6.26 | 11.41 |
| Big | 626 | 90.48 | 0.92 | 4.36 | 1.07 | 5.1 | 8.77 |
| All but micro | 1229 | 96.93 | 0.94 | 4.42 | 1.13 | 5.57 | 10.22 |

***Source: Fama and French (2005)***

**Table 3: Summary stats small company premium**

| Average | Monthly | Values, | July | 1963 | – | December | 2005 |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | Firms | % of total market cap | Ave | Std Dev | Ave | Std Dev | |
| Market | 3060 | 100 | 0.94 | 4.44 | 1.36 | 6.14 | 15.14 |
| Micro | 1831 | 3.07 | 1.29 | 6.84 | 1.56 | 6.99 | 17.51 |
| Small | 603 | 6.45 | 1.22 | 6.03 | 1.21 | 6.26 | 11.41 |
| Big | 626 | 90.48 | 0.92 | 4.36 | 1.07 | 5.1 | 8.77 |
| All but micro | 1229 | 96.93 | 0.94 | 4.42 | 1.13 | 5.57 | 10.22 |

***Source: LE elaboration of Fama and French data***

**Table 4: Summary stats small company premium**

| Average | Monthly | Values, | July | 1963 | – | December | 2005 |
|---|---|---|---|---|---|---|---|
| | | | | | | | |
| | Firms | % of total market cap | Ave | Std Dev | Ave | Std Dev | |
| Market | 3060 | 100 | 0.94 | 4.44 | 1.36 | 6.14 | 15.14 |
| Micro | 1831 | 3.07 | 1.29 | 6.84 | 1.56 | 6.99 | 17.51 |
| Small | 603 | 6.45 | 1.22 | 6.03 | 1.21 | 6.26 | 11.41 |
| Big | 626 | 90.48 | 0.92 | 4.36 | 1.07 | 5.1 | 8.77 |
| All but micro | 1229 | 96.93 | 0.94 | 4.42 | 1.13 | 5.57 | 10.22 |

***Source: LE elaboration of Fama and French data***

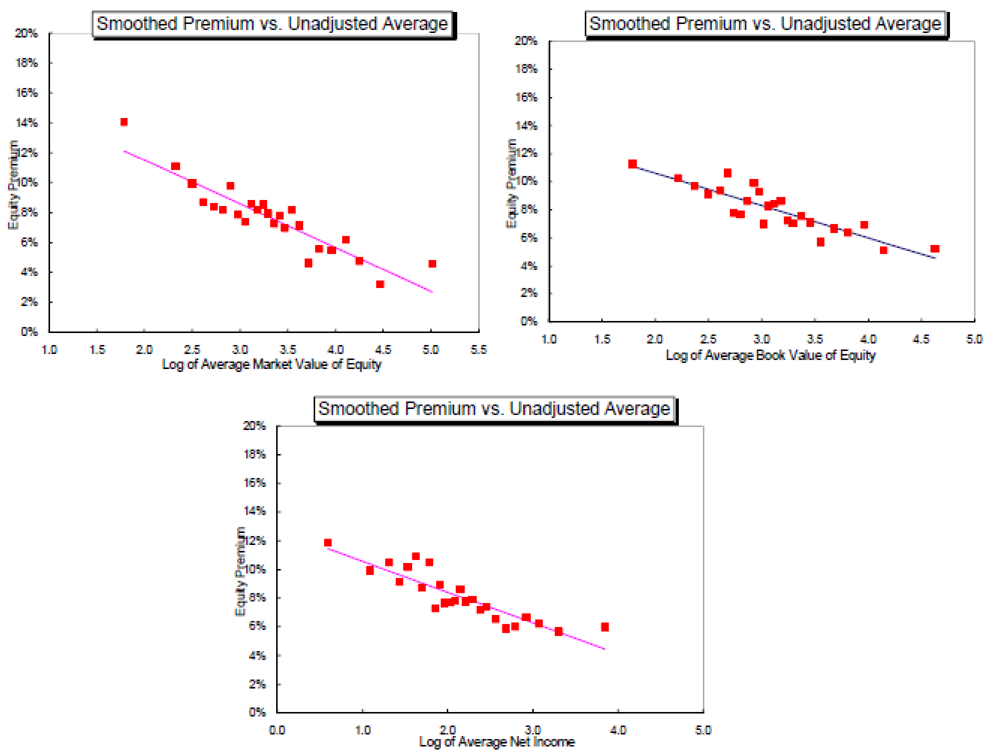
### 3.1.3 Small firms premium evidence

***Duff & Phelps, LLC (2010): Risk Premium Report***

In their Risk Premium Report of 2010, Duff & Phelps (LLC) use fundamental accounting measures in their analysis of the existence of a small cap premium. Using the Centre for Research in Security Prices (CRSP) database and the Standard & Poor's, Compustat database, a selection of publicly traded companies from 1963 to 2009 were chosen for analysis, controlling for selection bias and delisting bias. Using 8 alternative measures for size, portfolio breakpoints were chosen such that, in each instance, NYSE companies were divided into 25 even groups. With the breakpoints chosen, NASDAQ and AMEX companies were then added to the portfolios. Portfolios were rebalanced annually; with the portfolio return calculated using an equal-weighted average of the return on equities in the portfolio.

The figure below shows the relationship between measures of equity size and the return on equity from the ranked portfolios. Depending on the measure of equity size used, a 10% increase in equity size was found to reduce the average annual premium by between 0.22% and 0.29% per annum, while the difference between large stock and small stock. The Historical Average Debt to Market Value of Invested Capital is approximately 20% across all portfolios, indicating leverage levels do not account for the small firm premium.

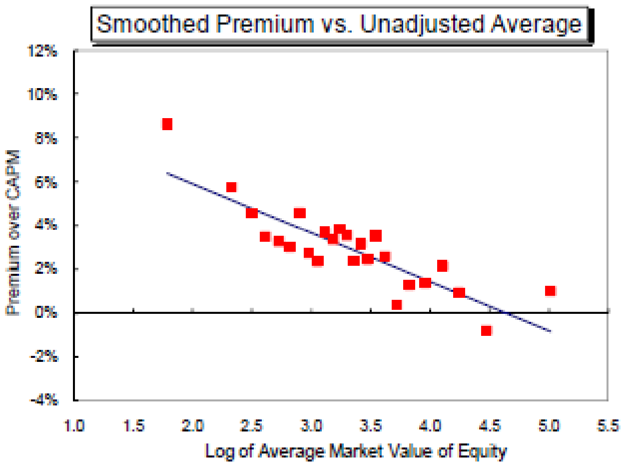
**Figure 3.2: Relationship between Equity Size and Returns, 1963-2009**




***Source Duff & Phelps Risk Premium Report (2010)***

Measuring the returns in excess of CAPM expectations as the size premium, Duff & Phelps again found a small firm premium across all measures of firm size. Examining the market capitalization results, they found that a 10% increase in the average market value of equity resulted in a 0.22% decrease in the premium over CAPM (significant at the 0.1% level), with an R-squared of 73%.

**Figure 3.3: Market Value of Equity-Based Returns in Excess of CAPM, 1963-2009**




***Source Duff & Phelps Risk Premium Report (2010)***

The table below presents data on the 25 portfolios used by Duff & Phelps, ranked by Market Value of Equity. As can be seen from the data, there is a strong, positive trend in the premium over CAPM, indicating a strong, positive small cap premium. Using the Fama and French methodology, subtracting the average premium over CAPM of the 12 large cap portfolios from the average premium over CAPM of the 12 small cap portfolios yields an average "small minus big" annual premium of 2.41%.

**Figure 3.4: Summary Statistics for Portfolios, Ranked by Market Value of Equity, 1963-2009**

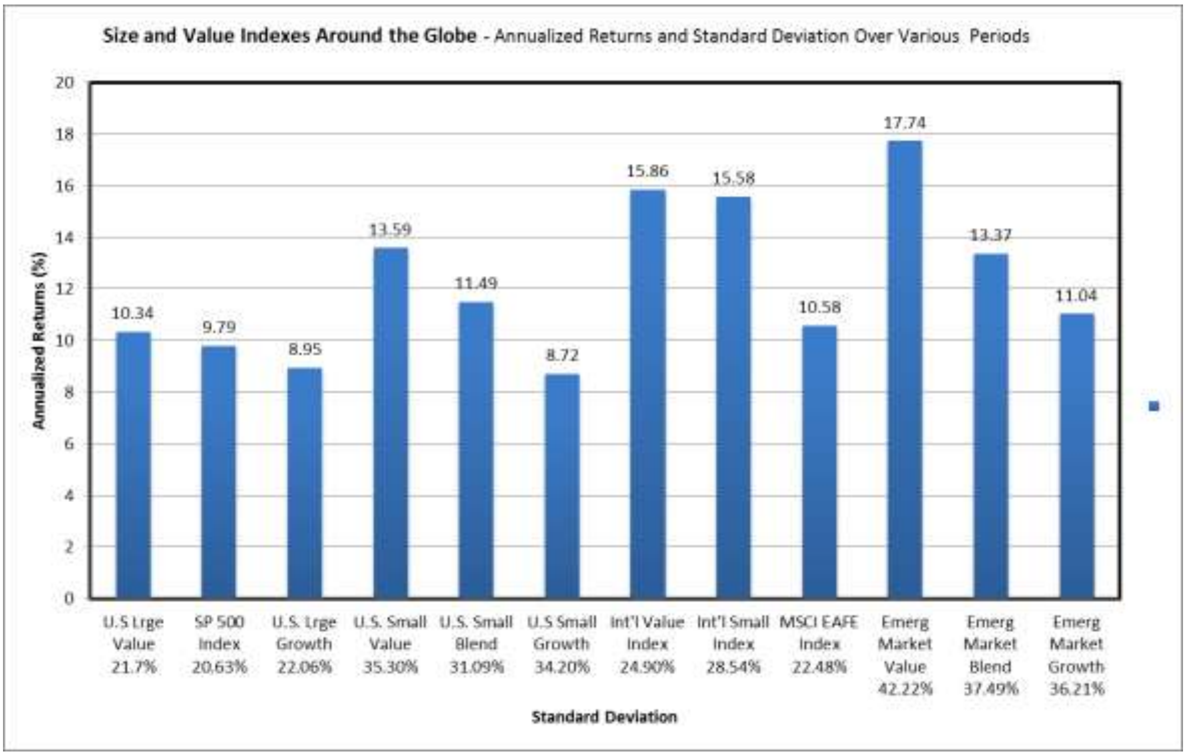
| Portfolio Rank | Beta | Average return | Average Risk Premium | CAPM Premium | Premium Over CAPM |
|---|---|---|---|---|---|
| 1 | 0.84 | 11.53% | 4.57% | 3.58% | 0.99% |
| 2 | 0.94 | 10.16% | 3.20% | 4.01% | -0.80% |
| 3 | 0.9 | 11.73% | 4.77% | 3.84% | 0.92% |
| 4 | 0.95 | 13.15% | 6.19% | 4.05% | 2.14% |
| 5 | 0.97 | 12.45% | 5.49% | 4.13% | 1.36% |
| 6 | 1.01 | 12.55% | 5.59% | 4.31% | 1.28% |
| 7 | 1 | 11.59% | 4.63% | 4.26% | 0.36% |
| 8 | 1.08 | 14.11% | 7.15% | 4.57% | 2.58% |
| 9 | 1.1 | 15.12% | 8.16% | 4.66% | 3.50% |
| 10 | 1.06 | 13.93% | 6.97% | 4.53% | 2.44% |
| 11 | 1.1 | 14.78% | 7.82% | 4.67% | 3.14% |
| 12 | 1.15 | 14.22% | 7.26% | 4.88% | 2.37% |
| 13 | 1.04 | 14.90% | 7.94% | 4.41% | 3.53% |
| 14 | 1.11 | 15.49% | 8.53% | 4.72% | 3.81% |
| 15 | 1.14 | 15.15% | 8.19% | 4.85% | 3.34% |
| 16 | 1.15 | 15.54% | 8.58% | 4.90% | 3.68% |
| 17 | 1.19 | 14.35% | 7.39% | 5.05% | 2.34% |
| 18 | 1.21 | 14.82% | 7.86% | 5.13% | 2.73% |
| 19 | 1.22 | 16.71% | 9.75% | 5.17% | 4.58% |
| 20 | 1.22 | 15.15% | 8.19% | 5.19% | 3.00% |
| 21 | 1.21 | 15.35% | 8.39% | 5.13% | 3.26% |
| 22 | 1.23 | 15.66% | 8.70% | 5.22% | 3.48% |
| 23 | 1.27 | 16.91% | 9.95% | 5.38% | 4.57% |
| 24 | 1.26 | 18.06% | 11.10% | 5.35% | 5.75% |
| 25 | 1.27 | 20.99% | 14.03% | 5.40% | 8.63% |

***Source Duff & Phelps Risk Premium Report (2010)***

### *Index Fund Advisors report*

There has been some concern by regulators as to whether the size premium effect occurs outside the USA. A number of reports and index fund advisors present evidence that in fact the size premium has tended to be bigger outside the USA. The below is from IFA, 2010 index fund report analysis. Small firms tend to have higher returns and higher standard deviations of returns, in the USA and internationally.

**Figure 5: International evidence on the size premium**




***Source: IFA***

### *Ibbotson Style Indices (2003): A Comprehensive Set of Growth and Value Data*

In the Ibbotson Style Indices (2003), Barard also uses a combination of the trimmed CRSP database of NYSE, AMEX and NASDAQ stocks, from 1964 to 2002, to construct four market capitalization-based portfolios (large-cap, mid-cap, small-cap, and micro-cap), with breakpoints at the 20th, 50th and 80th percentiles. Removing all companies with invalid book-to-price ratios, the portfolios were divided into “style” (value or growth) portfolios, resulting in a total of 8 portfolios, disaggregated by market cap and style, such that the total market cap of the growth and value indices are equal within each portfolio. For each year, portfolios were formed in June, with value-weighted monthly returns calculated from July to the following June.

Across all equities, value stocks were found to outperform growth stocks, with a compound return of 11% versus 8.8% per annum across the observed sample. Similarly, when disaggregated by market cap size, value portfolios were found to outperform growth portfolios across all 4 market cap sizes. Within the value portfolios, there was a negative correlation between portfolio return and market cap size. The table below shows the results of the analysis.

**Figure 3.6: Annual Returns by Value Portfolio, 1969-2002**

| | Return in 2002 from $1 in 1969 | Annualised Return | Standard Deviation | Sharpe Ratio |
|---|---|---|---|---|
| Micro-Cap Portfolio | 104.82 | 14.7% | 24.7% | 0.44 |
| Small-Cap Portfolio | 95.38 | 14.4% | 21.7% | 0.46 |
| Mid-Cap Portfolio | 64.36 | 13.0% | 19.4% | 0.42 |
| Large-Cap Portfolio | 29.15 | 10.4% | 17.0% | 0.31 |
| Aggergate Value Portfolio | 34.63 | 11.0% | 17.1% | 0.21 |

***Source: Ibbotson Style Indices (2003)***

Investment in the Micro-Cap value portfolio gives the highest return of any portfolio, with a compound return of 11.0% between 1969 and 2002. The Large-Cap portfolio gave the lowest return of any of the Value portfolios, with a compound annual return of 10.4%. Comparing Sharpe Ratios (a measure of excess return per unit of risk), the Small-Cap Value portfolio was found to give the greatest return-to-risk ratio, followed respectively by the micro-cap, mid-cap and large-cap value portfolios. Across all 4 portfolios, value portfolios were found to have a higher Sharpe Ratio than growth portfolios, while no growth portfolio had a higher Sharpe Ratio than any of the value portfolios, regardless of market cap size.

### 3.1.4 Evidence on the size premium in the UK and internationally

Until recently, one of the more open questions regarding the size premium was how likely the finding would hold in other markets. More recently, however, there has been a wave of studies confirming the size premium. In addition, the speed within which the data are updated has increased, so the available evidence is more up-to-date.

In a recent paper, Gregory et al (2009)[15], studied the UK FFTM and formed up-to-date portfolios sorted on size and a variety of factors. Their primary goal was to form a Fama and French 3-factor model database, and to update both the factors and the portfolios on a regular basis.

Gregory and Michou (2009)[16] also studied the three factor model and related models of UK asset returns on an industry basis. They also gave a presentation on the subject at a recent conference on the regulatory cost of capital, explaining their results. Overall, their test of several models is focused on which model performs best, but we focus on whether their evidence supports the idea of a small company premium. A few points they make are very relevant to our study.

Generally, Gregory and Michou would argue for longest possible run of data to estimate historical returns (or factor premia). Using the standard (static) Fama and French three factor methodology, they find for most industries (there are about 32):

---

15 Gregory, Alan, Rajesh Tharyan and Angela Huang Xfi (2009), “The Fama-French and Momentum Portfolios and Factors in the UK”, Centre for Finance and Investment, University of Exeter Paper No 09/05 December 2009.

16 Gregory, Alan, Maria Michou, “Industry Cost of Equity Capital: UK Evidence,” University of Exeter Xfi Centre for Finance and Investment, University of Exeter Business School and University of Edinburgh University of Edinburgh Business School Paper Number: 07/08 February 2009.

• 23 industries have significant positive loadings on SMB (in other words, a small premium effect) (We note that for gas and water utilities, the effect was positive too).

• 9 industries have significant positive loadings on HML

• adjusted R- square is around 34%

Gregory and Michou also tested a conditional CAPM and 3-factor model, but find that use of the conditional model is not justified: "For [the] conditional 3 FM, changes do not occur in [a] fashion predicted by changes in size and B/M, so unlike [the] US [there] seems to be little evidence to justify using a conditional model."

They conclude that the 3-factor model "has some merit" and is marginally better than other models (such as plain CAPM), and significantly better than others (4-factor models), but they feel the results are disappointing in that there remains a large portion of future returns which are not explained. They therefore conclude that more research is needed. It would be interesting to see if dividing their industry portfolios on size created any statistically significant size impacts based on a simple test on the statistical significance of the SMB coefficient.

In another recent paper, Cuthberston et. al. (2008)[17] study various models which explain UK returns. (They focus on mutual funds, so they do not form their own portfolios). Interestingly, they correct for previously posed "problems" in the original 3-factor models, such as the possibility of survivorship bias. Nonetheless, they find positive and significant SMB coefficients on the small factor (the SMB beta). Their data for regression results are found in the figure below.

Table 2
Model selection: cross-sectional results of model estimations

| Model | 1 | 2 | 3 |
|---|---|---|---|
| | Unconditional | Conditional-beta | Conditional alpha and beta |
| Regression coefficients | | | |
| Average alpha (percent p. m.) | −0.057 | −0.032 | −0.109 |
| Standard deviation of alpha | 0.261 | 0.261 | 0.754 |
| Unconditional-betas (*t*-stats in parentheses) | | | |
| $r_{mt}$ | 0.912 (25.196) | 0.863 (21.193) | 0.849 (21.068) |
| $SMB_t$ | 0.288 (4.959) | 0.285 (4.905) | 0.257 (4.043) |
| $HML_t$ | −0.025 (1.451) | −0.023 (1.451) | 0.016 (1.207) |
| Conditional variables, $Z_{t-1}$ (dividend yield) | | | |
| $z_{t-1}r_{mt}$ | – | −0.048 (1.408) | −0.055 (1.496) |
| $z_{t-1}SMB_t$ | – | – | −0.002 (1.513) |
| $z_{t-1}HML_t$ | – | – | 0.033 (1.044) |
| $z_{t-1}$ | – | – | −0.073 (1.037) |
| Model selection criteria | | | |
| Adjusted *R*-squared | 0.807 | 0.811 | 0.818 |
| SIC | 1.352 | 1.365 | 1.432 |
| Residuals not normally distributed (% of funds) | 64 | 64 | 63 |
| LM(1) statistics (% of funds) | 40 | 40 | 44 |
| LM(6) statistics (% of funds) | 34 | 35 | 39 |
| Equally weighted portfolio | | | |
| Alpha | −0.139 | −0.112 | −0.107 |
| *t*-statistics | −3.051 | −2.464 | −2.517 |

The SMB coefficient (SMB-beta) is found to be circa 0.288 and not sensitive to conditioning on other variables in the unrestricted sample.

[17] Cuthbertson, Keith ,Dirk Nitzsche, Niall O'Sullivan (2008), "UK mutual fund performance: Skill or luck?", Journal of Empirical Finance 15 (2008) 613–634.

In the table below they also test whether the effect is there for the best and worst funds (the focus of their paper is to test fund manager performance). Here they find similar results for the SMB beta.

Table 6
Kalman filter: fund of funds (July 1985–December 2002)

| | | Portfolio of best 12 funds | | | Portfolio of worst 50 funds | | |
|---|---|---|---|---|---|---|---|
| | | Final state vector | p-value | Standard error time varying parameter | Final state vector | p-value | Standard error time varying parameter |
| Constant | $\alpha_T$ | 0.4463 | 0.0083 | 0.0279 | −0.3379 | <0.0001 | 0.0003 |
| $r_m$ | $\beta_{1T}$ | 0.9077 | <0.0001 | <0.0001 | 0.9465 | <0.0001 | 0.0007 |
| SMB | $\beta_{2T}$ | 0.3168 | 0.0001 | 0.0189 | 0.2335 | <0.0001 | 0.0004 |
| HML | $\beta_{3T}$ | −0.1241 | 0.0573 | 0.0157 | −0.0339 | 0.2192 | 0.0004 |
| Log likelihood | | −348.33 | | | −249.35 | | |

They conclude, "The market excess return and the [size] SMB factor betas are consistently found to be statistically significant across all three classes of model, whereas the [value] HML factor beta

***Dimson, Nagel and Quigley (2003): Capturing the Value Premium in the UK***

The size premium effect has been documented for the UK.

Dimson, Nagel and Quigley (2003), used equity data from the London Share Price Database, to replicate the Fama and French (1993) three factor model results in the UK market. The LSPD data was linked with fundamental data from Datastream, Cambridge DTI and stock exchange yearbooks, for the period 1955-2001 to establish the existence of a small cap and a value premium. As the database contained non-surviving companies and almost complete data for all listed companies since 1953, there was no survivor bias present in the data.

Portfolios were formed according to market capitalization and value, where size groupings were determined using the market value of equity with a 70$^{th}$ percentile breakpoint, while value groupings were constructed using the book to market equity ratio and breakpoints at the 40$^{th}$ and 60$^{th}$ percentile. Value weighted returns for each of the portfolios were calculated for a 12 month buy-and-hold period. If an equity was delisted during the holding period, proceeds from the equity were distributed among the other equities in the portfolio according to their value weighting.

The table below presents summary statistics from the set of constructed portfolios. There is clear evidence of a small cap premium, independent of any value premium, with small cap portfolios outperforming large cap portfolios across all value groupings.

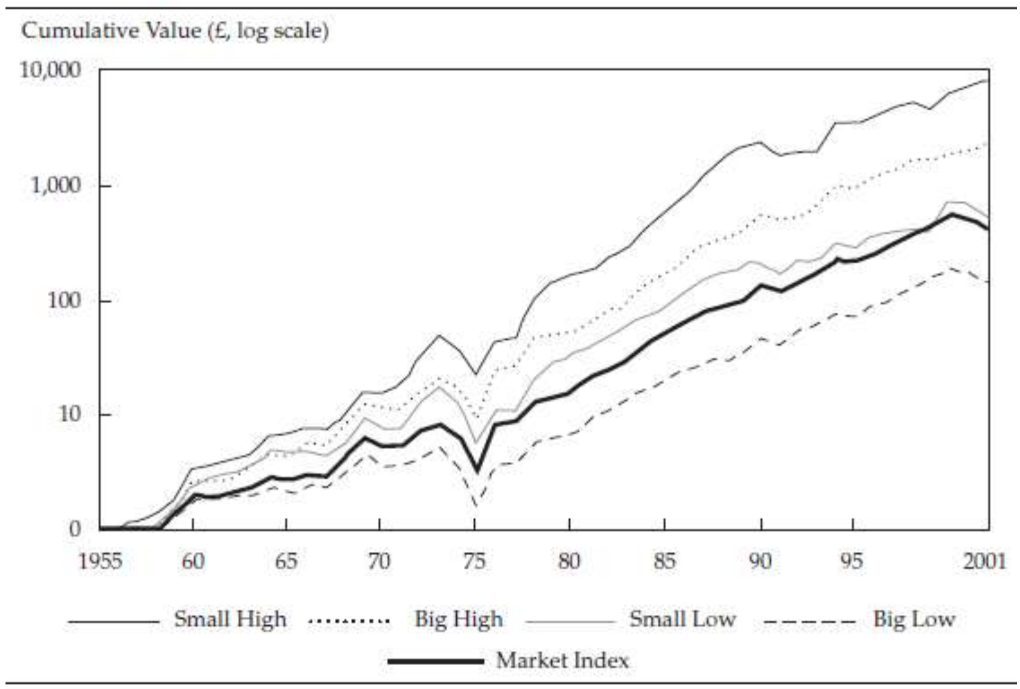
**Figure 3.7: UK Portfolio Summary Statistics, 1955-2001**

| | Average Monthly Return | | Standard Deviation | |
|---|---|---|---|---|
| | Small Cap | Big Cap | Small Cap | Big Cap |
| Low | 1.26% | 1.06% | 5.14% | 5.70% |
| Medium | 1.52% | 1.47% | 4.91% | 5.65% |
| High | 1.74% | 1.56% | 4.77% | 5.84% |

***Source: Dimson, Nagel and Quigley (2003)***

Dimson, Nagel and Quigley also compared the cumulative performance of the six portfolios with a UK value weighted market index. The results are presented in the graph below, which represent a hypothetical £1 investment in each of the portfolios in July 1955, with dividends reinvested in the index constituents.

**Figure 3.8: Cumulative Return to Portfolios, July 1955 to December 2001**




***Source: Dimson, Nagel and Quigley (2003)***

They conclude that the size premium is a significant determinant of returns in the UK, but that capturing these returns for fund managers could be a problem due to illiquidity of the smallest firms.

The importance of the Dimson, Marsh and Quigley study should be emphasized. The long-term perspective shows very clearly that small caps in the UK, similar to elsewhere, have earned a premium return in the long run. From the chart (view the slopes of the lines, and recall the vertical scale is in logs), over periods of even a decade small caps may have underperformed a variety of large caps, such as in the 1990s. But over the 1955 to 2001 period the overall average return to small caps is much higher.

Others have studied the size and value factor premium using international data. Maroney and Propapadakis (2002), estimated FFTM for each of Australia, Canada, France, Germany, Japan, the UK and the USA, and find the SMB and HML factors to be significant and as expected in all markets.

### 3.1.5 Additional research on size, liquidity, and equity returns

One of the arguments levelled against the theory of the Small Cap premium is that capitalization-weighted indexing over-weights temporarily overvalued equities and underweights temporarily undervalued equities. Under this argument, indices that are weighted using fundamental factors (or a combination of multiple fundamental factors) such as earnings, dividends, sales and book value, which better represent the intrinsic value of the firm and achieve superior risk-adjusted returns than capitalization-weighted indexing as they do not undervalue/overvalue equity. However, there is increased risk in applying too much weight to value stocks (stocks with a high book-to-market ratio) and too little weight to growth stocks (stocks with high earnings growth and high profitability ratios).

To test the benefits of fundamental indexing over capitalization-weighted indexing, Jun and Malkiel (2008) run a hypothetical backtest on the comparative performance of a Fundamental Index and an S&P 500 Index and a hypothetical portfolio (33% S&P EWI 33% Midcap Value, and 33% Russell 1000 Value indices) between 2000 and 2007. Their results are presented in the table below.

**Figure 3.9: Hypothetical Portfolio Backtest, Comparative Performance 2000-2007**

| | FI | S&P 500 | Portfolio |
|---|---|---|---|
| Annualized Returns | 8.10% | 1.59% | 8.79% |
| Monthly Volatility | 3.84% | 3.98% | 4.08% |
| Sharpe Ratio | 1.27 | -0.42 | 1.36 |

***Source: Jun and Malkiel (2008)***

From these results, combined with longer-term evidence of mean reversion in the outperformance of value stocks, Jun and Malkiel conclude that the performance of the fundamentally weighted index can be explained by Fama-French risk factors of size and value, supporting the theory of the small capitalization premium.

In a 2010 MCSI Research Insight paper, Kilbert and Subramanian argue that the small cap equity risk premium is still in existence, that small cap stocks rebounded faster than large cap stocks post 2008, and that passive investment strategies can capture the small cap premium. Using data from 1994 to 2010, Kilbert and Subramanian show that the MSCI World Small Cap stocks outperform the MSCI World Large and Mid-Cap Stocks, with an annualized return of 6.7% versus 5.5%, albeit with a higher standard deviation. Using passive indexing strategies (ranging from full replication of the MSCI World Small Cap Index, to portfolios constructed using optimization techniques and the Barra Global Equity Model) it is shown that small cap portfolios will capture a risk premium.

There is a large literature on capital market imperfections arising from asymmetric information and incentive problems, and the constraints they place on firms' ability to access credit.

We have identified two broad strands to this literature: one focusing on adverse selection (see Stiglitz and Weiss, 1981); and the other focusing on moral hazard (see Jaffee and Russell, 1976; and Piketty, 1997) as sources of capital market imperfections.

Given these features, capital owners are unable to accurately determine the creditworthiness of firms and reflect this in the terms on which credit is extended. For instance, capital owners may prefer to extend credit with greater collateral requirements that compensates them in the event of default.

As a consequence, firms are constrained in their ability to finance working capital and investments in fixed capital. However, smaller firms are likely to be particularly affected because, among other things, they possess fewer forms of eligible collateral.

Empirically, a review of evidence on credit constraints faced by SMEs is provided by Beck and Demigruc-Kunt (2006), with macro-level studies having been conducted by Berger and Udell (1998) and recent, sophisticated micro-level work having been undertaken by Banerjee and Duflo (2008) and De Mel, McKenzie and Woodruff (2008).

***Liquidity premia***

Another area of research that is related to the size premium is the idea of a liquidity premium. Some researchers have in fact hypothesized that the FFTM SMB size effect is in fact a liquidity effect that is proxied by size.

London Economics (2002) studied the impacts of reducing liquidity premia (as measured by trading-bid-ask spreads) for UK and EU major stock markets for the EU Commission. The study has recently been updated for the UK (London Economics 2010).[18] For the purposes here, the main point of those studies is that market structure and a variety of factors tends to significantly impact liquidity, and also that the market structure and degrees of financial market integration can have significant impacts on the cost of equity capital, either via fees, via bid-ask spreads on trades, or on other variables.

***Time period selection***

Another consideration of estimation of risk premia in general is their potential sensitivity to the time period selected. In fact, the most common approach is to use the longest time series available, although sensitivity analysis suggests our results are not particularly sensitive to the time period selected. An alternative approach is to use a filtration statistical approach so as to not allow recent random events to impact the overall statistics of the series too greatly (we did this for the USA data FFTM). Finally, it is noteworthy that recent financial events of the crisis, if included, will have large impacts on potential risk premia estimated.

### 3.1.6 Rebuttal of suggestion that the size premium didn’t exist or had disappeared

***Rebuttal that size premium doesn’t exist***

We have found a number of examples of UK regulator’s and consultants reports which consider that the size premium does not exist in the UK, in spite of the fact that over the long run small caps in the UK have outperformed large caps by so wide a margin. These conclusions usually are based on the following type analysis, where company returns are regressed on market factors, such as a small minus big (SMB) factor, and then the SMB coefficient is interpreted as whether it is significantly different from zero.

A particular example is work provided for Ofwat[19], which is found in the table below.

[18] London Economics (2010), “Understanding the future impacts of MiFID,” A report for the City of London. Available at www.londecon.co.uk.

[19] Available at: http://www.ofwat.gov.uk/pricereview/pr09phase3/rpt_com_20091126fdcoc.pdf

Table 9.9: Fama and French estimated coefficients

| Company | $\beta_A$ | $s_A$ | $h_A$ | Number of observations |
|---|---|---|---|---|
| United Utilities | 0.54** | -0.1 | 0.22 | 185 |
| Severn Trent | 0.38* | 0.003 | 0.30 | 173 |
| Pennon | 0.37* | 0.001 | 0.34 | 185 |
| Kelda | 0.42* | -0.04 | 0.33 | 173 |
| Northumbrian | 0.73 | -0.34 | -0.17 | 31 |
| Dee Valley | -0.05 | 0.19 | 0.17 | 132 |
| East Surrey | 0.22* | -0.17 | 0.09 | 163 |
| Bristol Water | 0.20** | 0.15 | 0.18 | 144 |

*Source: Europe Economics estimations from Bloomberg data and Risky Style database. ** significantly different from zero at 99 per cent level of confidence, * significantly different from zero at 95 per cent level of confidence.*

The table above tells us that the coefficient on the SMB factor is small and not statistically significant for all of the companies listed. For two of the companies, the CAPM-market beta is not significant (Dee Valley and Northumbrian), while for only Bristol and United Utilities is the CAPM beta significant to a high degree of confidence (99%). The biggest criticism of the above table as evidence as to whether the size premium exists *is whether the companies above are small—they are not (save East Surrey and Dee Valley).* The company sizes are found in the table below.

**Table 5: Sizes of listed UK water companies -- total company assets from balance sheet**

| Company Name | Asset value £billions |
|---|---|
| Pennon | £3.9 |
| Severn Trent | £7.4 |
| United Utilities | £8 |
| Kelda | £4.9 |
| Northumbrian | £3.9 |
| Dee Valley | £0.092 |
| East Surrey | £0.172 |
| Bristol | £0.339 |

***Source: LE from company reports/balance sheets***

By most definitions of a small company, these are not small. United Utilities is in the FTSE 100. It is the expected sign and outcome that the premium could be negative or insignificant.

Defining "smallness" is a relative thing, but it is clear from comparing the table above with data from the LSE (previously reported in figure 1), the top companies would be in the FTSE 100, 250, or 350—so in general big companies. Relative to the IGT sector, East Surrey and Dee Valley are exceptions (i.e., are small companies), and it might be considered Bristol is borderline (in addition, we note that we are comparing total assets with market caps for the LSE data)[20].

[20] We note we are only comparing to the IGT sector.

However, when one considers small companies, and wishes to test in a valid way the FFTM, more detail and more effort is typically needed for best practice. In considering additional information needed to judge the FFTM versus the Sharpe-Lintner CAPM, such as the above table in the report for Ofwat, we would look to the following:

1) What the R-squared and F-statistics for the above models vìs-a-vìs the CAPM?
2) A better test of the FFTM would be a likelihood ratio test of the nested (Sharpe-Lintner CAPM) versus the FFTM.
3) The definition of the SMB factor should be consistent with the definition of a small company given the company in question (e.g., if the SMB factor is defined as small minus big for a break point of 'small' defined on market cap of say £500m, then this should be consistent with the company size, which might be circa £400m).
4) A further consideration should be given to the liquidity of the shares and whether the CAPM is fitting or a workable model for the small company first.
5) The small number of observations of Northumbrian makes it of little use in judging the FFTM, as there are at least 5 parameters to estimate, and only 31 observations.

***Testing the FFTM with some UK utility share price data***

Similar results (i.e., one will find [not surprisingly] big companies don't get a small company premium) can be had by regressing, say, Centrica returns on the UK FFTM factors. These results are presented below.

The results show a somewhat low R-squared, at 17%. The number of observations is limited. The HML factor is not significant. The SMB factor is marginally significant but negative. These results are what one would expect since Centrica is a BIG company, and it is not under distress or a value stock (high-book-to-market). Comparing with the CAPM, the FFTM does not seem to be adding much. So the conclusion is when a company is big and not a value stock, it makes little difference which model is used, so CAPM is fine.

**Figure 10:Centrica FFTM vs Sharpe-Lintner CAPM**

```
. reg   cna_bbr_rf  rmrf smb hml

      Source |       SS       df       MS              Number of obs =     142
-------------+------------------------------           F(  3,   138) =    9.63
       Model |  .127594842     3  .042531614           Prob > F      =  0.0000
    Residual |   .60974312   138  .004418428           R-squared     =  0.1730
-------------+------------------------------           Adj R-squared =  0.1551
       Total |  .737337962   141  .005229347           Root MSE      =  .06647


------------------------------------------------------------------------------
  cna_bbr_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rmrf |   .5745549   .1442473     3.98   0.000     .2893342    .8597755
         smb |  -.2301735   .1383754    -1.66   0.099    -.5037836    .0434366
         hml |  -.0621778   .1354383    -0.46   0.647    -.3299805    .2056248
       _cons |   .0136513   .0057884     2.36   0.020     .0022059    .0250967

. reg   cna_bbr_rf  rmrf

      Source |       SS       df       MS              Number of obs =     142
-------------+------------------------------           F(  1,   140) =   25.86
       Model |  .114966568     1  .114966568           Prob > F      =  0.0000
    Residual |  .622371394   140   .00444551           R-squared     =  0.1559
-------------+------------------------------           Adj R-squared =  0.1499
       Total |  .737337962   141  .005229347           Root MSE      =  .06667


------------------------------------------------------------------------------
  cna_bbr_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rmrf |   .6727528   .1322911     5.09   0.000     .4112063    .9342994
       _cons |   .0111467   .0055953     1.99   0.048     .0000844    .0222089
------------------------------------------------------------------------------
```

***Source: LE***

We run a similar test on Scottish and Southern (SSE), which is also a big company. The model confirms a negative and significant small company premium, as one would expect for a big company. The hml factor is significant and positive. The R-squared value is low.

**Figure 11:SSE FFTM**

```
. reg     sse_bbr_rf   rmrf smb hml

      Source |       SS       df       MS              Number of obs =     192
-------------+------------------------------           F(  3,   188) =    8.53
       Model |  .105138798     3  .035046266           Prob > F      =  0.0000
    Residual |  .772381127   188   .00410841           R-squared     =  0.1198
-------------+------------------------------           Adj R-squared =  0.1058
       Total |  .877519925   191  .004594345           Root MSE      =   .0641

------------------------------------------------------------------------------
  sse_bbr_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rmrf |   .3103537   .1255408     2.47   0.014      .062704    .5580033
         smb |  -.2765027   .1133193    -2.44   0.016    -.5000434    -.052962
         hml |   .3158033   .1254639     2.52   0.013     .0683054    .5633012
       _cons |   .0110857   .0049118     2.26   0.025     .0013963    .0207751
------------------------------------------------------------------------------
```

***Source: LE***

Finally, we consider a small recently listed electric company (the time period is 31/10/2005 to end 2008-where our factor data ends), IPSA, who is involved in IPP. In the case of IPSA, the SMB factor is the *only* coefficient that is significant, and the standard CAPM does little to explain the share's movements and total returns. The CAPM regression is of little value. We note that the small number of observations available is an issue. The likely conclusion is probably that CAPM or the FFTM are not valid for this company until more data are available. It is also possible that illiquidity is driving some of these results. The regression results are found below.

**Figure 12:IPSA Electric FFTM vs Sharpe-Lintner CAPM**

```
.reg     ipsa_bb_rf rmrf smb hml

      Source |       SS       df       MS              Number of obs =      39
-------------+------------------------------           F(  3,    35) =    1.94
       Model |  13.1915737     3  4.39719124           Prob > F      =  0.1412
    Residual |  79.3615209    35  2.26747202           R-squared     =  0.1425
-------------+------------------------------           Adj R-squared =  0.0690
       Total |  92.5530946    38  2.43560775           Root MSE      =  1.5058

------------------------------------------------------------------------------
   ipn_bb_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rmrf |   .1928199   8.066692     0.02   0.981    -16.18343    16.56907
         smb |   17.37229   9.376078     1.85   0.072    -1.662164    36.40674
         hml |   16.44839   15.54998     1.06   0.297    -15.11976    48.01653
       _cons |   .3896137   .2818283     1.38   0.176    -.1825282    .9617557
```

```
. reg     ipsa_bb_rf rmrf

      Source |       SS       df       MS              Number of obs =      39
-------------+------------------------------           F(  1,    37) =    0.01
       Model |  .017579737     1  .017579737           Prob > F      =  0.9336
    Residual |  92.5355148    37  2.50095986           R-squared     =  0.0002
-------------+------------------------------           Adj R-squared = -0.0268
       Total |  92.5530946    38  2.43560775           Root MSE      =  1.5814

------------------------------------------------------------------------------
   ipn_bb_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rmrf |   .4900434   5.844962     0.08   0.934    -11.35297    12.33306
       _cons |   .0525588   .2553829     0.21   0.838    -.4648962    .5700137
------------------------------------------------------------------------------
```

*Source: LE*

Consider the regression of the total monthly returns to Jersey Electric (small but listed UK utility) on the UK Fama and French three factors (e.g., the Gregory et al 2009 data). The liquidity issue is very real as is evident from the regression data below (and our more detailed look at the trading data for Jersey Electric). We present the regression results below; the first model is the FFTM and the second the CAPM. The F-statistic is close to one, so the total validity of the regression is not significant. The R-squared is very low, and none of the coefficients are significant. The model as estimated should not be a rejection of the FFTM—CAPM doesn't do any better. The results simply tell us that the returns series of Jersey Electric has little relationship on traded markets and traded factors. Further study of the returns data for Jersey Electric shows the share does not trade (price does not move) in many periods, indicating illiquid trading.

We also consider that the finding that an illiquidly-traded firm is not correlated with the market does not necessarily tell one that the firm's risk is diversifiable; the actual risks of the firm *may* in fact be well correlated with the market, but illiquidity means the risks of the firm are not necessarily showing up in the share price movements. All one can conclude is merely that the share price is not correlated with the market returns

## Figure 13:Jersey Electric FFTM vs Sharpe-Lintner CAPM

```
. reg  jerselec_bbr_rf  rmrf smb hml

      Source |       SS       df       MS              Number of obs =     178
-------------+------------------------------           F(  3,   174) =    0.40
       Model |  .001655125     3  .000551708           Prob > F      =  0.7526
    Residual |  .239545961   174  .001376701           R-squared     =  0.0069
-------------+------------------------------           Adj R-squared = -0.0103
       Total |  .241201086   177  .001362718           Root MSE      =   .0371

------------------------------------------------------------------------------
jerselec_b~f |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rmrf |  -.0184304    .076285    -0.24   0.809    -.1689935    .1321326
         smb |   .0273953   .0730776     0.37   0.708    -.1168373     .171628
         hml |  -.0671348   .0740976    -0.91   0.366    -.2133805    .0791109
       _cons |   .0046526   .0028863     1.61   0.109    -.0010441    .0103493
------------------------------------------------------------------------------


. reg  jerselec_bbr_rf  rmrf

      Source |       SS       df       MS              Number of obs =     178
-------------+------------------------------           F(  1,   176) =    0.17
       Model |  .000227683     1  .000227683           Prob > F      =  0.6839
    Residual |  .240973403   176  .001369167           R-squared     =  0.0009
-------------+------------------------------           Adj R-squared = -0.0047
       Total |  .241201086   177  .001362718           Root MSE      =    .037

------------------------------------------------------------------------------
jerselec_b~f |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        rmrf |  -.0281955   .0691422    -0.41   0.684    -.1646501    .1082591
       _cons |     .00468   .0027742     1.69   0.093     -.000795     .010155
------------------------------------------------------------------------------
```

***Source: LE***

***Rebuttal that size premium disappeared***

Over the course of the last few years, there has been some discussion in the academic and finance community that the size premium might have disappeared.

In general, however, we find there is little evidence to support this, as our detailed investigation has a) studied the breadth of the published research, and b) used a long-term time perspective along with best practice empirical methods.

When studying risk premia for regulatory purposes, it is generally accepted and believed that a long-term approach is necessary. The main reason for this is conclusions based on short histories are likely to be spurious. Choosing a time period of 20 years or less, especially if some filtration method is not applied, may not be statistically significant.

In addition, our strong opinion and judgment is that the published research on the notion that the size premium had disappeared was merely an artefact of a particular period in returns, e.g., the impact of the late boom, and the downturn. Effectively, the publication of the research on small caps led investors to look at small caps more closely, bidding up their prices. At the same time, in the late 2000s, large caps were overvalued, leading investors to seek other places for higher returns. However, the financial crisis then led investors to exit small caps and seek safer companies, reducing the return for small caps (for the short term average) even further. However, our empirical evidence has shown that even when including the most recent events, but when including a sufficiently long time series, the small cap premium remains.

Nonetheless, as some recent regulatory decisions in the EU have considered arguments against the small company premium and included what we believe is often either a misunderstanding of the main rationale of the multifactor models or selective use of evidence, we believe it is useful to rebut some of the arguments against the small company premium.

One argument against the small company premium suggests this is not consistent with the CAPM approach, which is in general favoured by UK and International Regulators, for example, in CER10103(j(1) Appendix 1 it states, "Clearly, the inclusion of a small company premium represents a departure from the CAPM, in which expected returns depend only on the systematic risk exposure of investors and not on the size of the company raising finance."

In our opinion, the Fama and French three factor model (FFTM) is 100% consistent with the notion of systematic or non-diversifiable risk and the CAPM, it merely says that a single factor (market excess returns) doesn't do as good a job at explaining returns as the 3-factor model. The whole point of the model is that the exposure to small company non-diversifiable risk is estimated by the beta coefficient on the SMB factor, while including the market excess return as an additional factor. Thus, if my portfolio has small caps, and if this risk is correlated with the SMB factor (small company excess returns), then this gets the appropriate weighting.

The same document (CER10103(j(1)), then also states, "For the period since 1981, however, there appears to be no small companies premium —it seems to have disappeared as soon as it was discovered (for the period 1981-2007, the geometric average annual small company effect was 0.08 per cent); indeed, during the 1990s there was a "small companies discount" (geometric average: -2.1 per cent). The current state of play is that there is very widespread doubt as to whether such an effect exists at all."

First of all, this so-called "widespread doubt" is pure conjecture. We have interviewed circa 10 equity analysts who are personal contacts of Dr Greg Swinand, Divisional Director at London

Economics. Among equity analysts who practice this for a living, there is no dispute that there is a small company premium, and that it has increased since the financial crisis.

Secondly, as our empirical evidence and filtration methodology have shown in this section, the result that the premium disappears is highly sensitive to the time period chosen. The notion that one can dispense with "premia" because they are sensitive to the time period chosen is in our opinion incorrect. Naturally, if one chooses a certain time period, given the random walk nature of share prices, then one is likely to find spurious trends and correlations—and naturally premia can be empirically negative in the short run. For example, The Economist recently ran an article noting that if one takes short run equity returns from almost anywhere in the last 5-10 years, one will get a very low or even negative market equity risk premium, as the financial crisis led to such large drops in share prices in late 2008/early 2009, that the average over a significant period is brought down. But taking a longer term perspective eliminates this problem. Nonetheless, there are no practitioners who would actually then conclude that the equity risk premium has fallen as a result of the financial crisis.

Some other considerations as to why the small company premium may exist include survivorship bias. We note that the studies by Duff and Phelps have included a detailed methodology to correct for this as have a number of other studies.

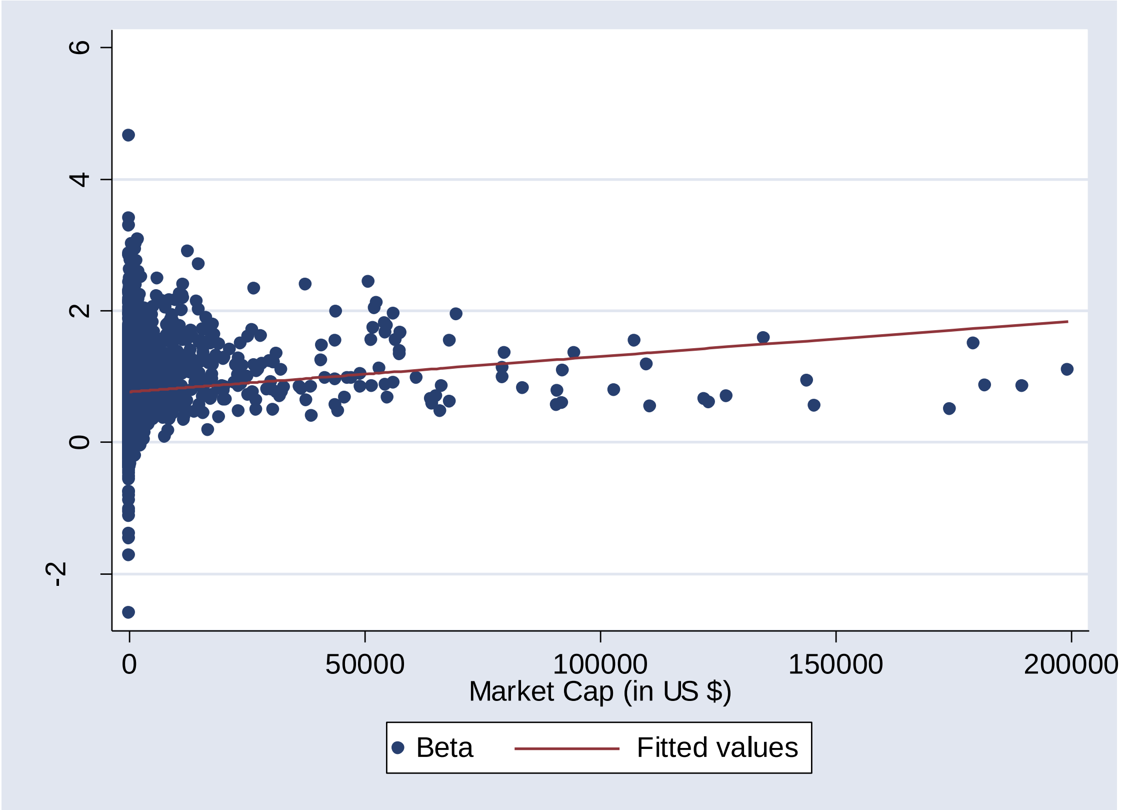
An alternative hypothesis is that the small company premium is a function of what is in fact an illiquidity premium. We believe the illiquidity premium is an important issue which deserves more research. For example, some studies have found that other measures of firm size seem to do a poor job of improving on the CAPM. If this is true, then it is of little import for the case at hand, because the companies (IGT) are both small, and not traded. In other words, while it might be of academic interest whether the 'true' premium is illiquidity or size, for our purposes we are only interested in whether the small companies should get a premium.

## 3.2 Evidence CAPM does not capture some size-related impacts for UK firms' betas

In this section, we conduct regressions on UK company betas. The betas are the dependent variables in the models. The data are betas from Bloomberg Data Professional Data terminal service, and the methodology is OLS on weekly returns for the last three years. We chose to download all the betas for UK listed companies and their associated basic market and accounting data, such as market cap, book values of equity and debt, etc.

We then regressed the values of beta on variables such as market cap, total cap (debt plus equity market value), as well as gearing and other risk ratios. The results showed that in general, even when comparing for other company risk variables, market cap variation still explained a significant amount of the variation in the betas. This is suggestive that the CAPM is not catching all of the risk, and the regressions indicate that betas for small companies might be (erroneously) depressed.

**Figure 14:Evidence 1**




Source: LE Analysis BB data

**Figure 15:Evidence 2**

```
. reg beta marketcapinus

      Source |       SS       df       MS              Number of obs =    4388
-------------+------------------------------           F(  1,  4386) =   59.67
       Model |  16.1233041     1  16.1233041           Prob > F      =  0.0000
    Residual |  1185.17094  4386  .270216812           R-squared     =  0.0134
-------------+------------------------------           Adj R-squared =  0.0132
       Total |  1201.29424  4387  .273830463           Root MSE      =  .51982

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
marketcapi~s |   5.37e-06   6.96e-07     7.72   0.000     4.01e-06    6.74e-06
       _cons |   .7693702   .0080315    95.79   0.000     .7536244    .7851159
```

***Source: LE Analysis***

In the first set of regression results, there is a significant coefficient on market cap, but the r-squared is very small, suggesting omitted variables. We therefore added historical average gearing levels, and market to book (value) ratio data as regressors. The results are below and improved.

**Figure 16:Evidence 3**

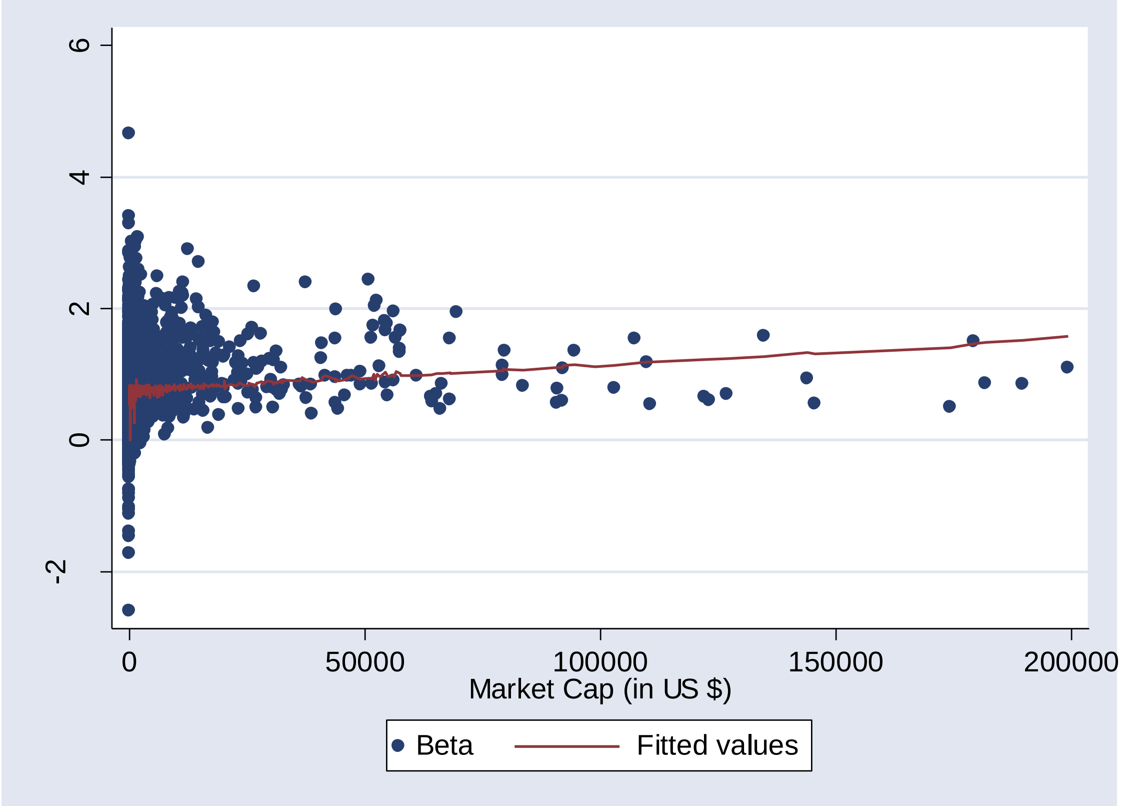
```
. reg beta marketcapinus historicalgrowthinnetincomelast3 mkt_book

      Source |       SS       df       MS              Number of obs =    1748
-------------+------------------------------           F(  3,  1744) =   19.94
       Model |  10.8879891     3  3.62932971           Prob > F      =  0.0000
    Residual |   317.44701  1744  .182022368           R-squared     =  0.0332
-------------+------------------------------           Adj R-squared =  0.0315
       Total |  328.334999  1747  .187942186           Root MSE      =  .42664

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
marketcapi~s |   3.88e-06   6.24e-07     6.22   0.000     2.66e-06    5.11e-06
historical~3 |  -.0950263   .0230755    -4.12   0.000    -.1402848   -.0497678
    mkt_book |  -.0000974    .000214    -0.46   0.649    -.0005172    .0003223
       _cons |   .7529269   .0107169    70.26   0.000     .7319076    .7739461
```

***Source: LE Analysis***

**Figure 17:Evidence 4**




***Source: LE Analysis***

Next, we added regressor data on book debt to equity ratio. This also is significant but the model still has only a small amount of explanatory power. Market cap still explains a statistically significant portion of the variation in the betas.

## Figure 18:Evidence 5

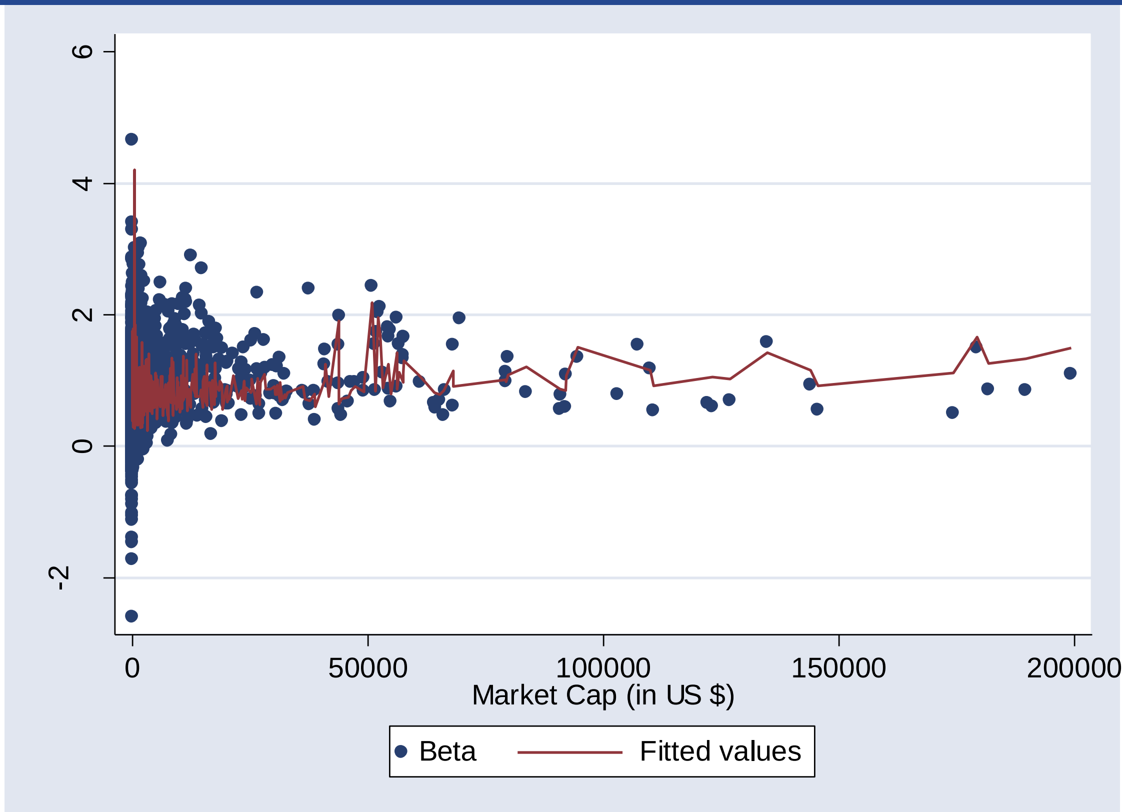
```
. reg beta marketcapinus bookdebttoequityratio
historicalgrowthinnetincomelast5 mkt_book

      Source |       SS       df       MS              Number of obs =    1338
-------------+------------------------------           F(  4,  1333) =   13.76
       Model |  9.51735809     4  2.37933952           Prob > F      =  0.0000
    Residual |  230.494965  1333  .172914453           R-squared     =  0.0397
-------------+------------------------------           Adj R-squared =  0.0368
       Total |  240.012323  1337  .179515575           Root MSE      =  .41583

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
marketcapi~s |   4.18e-06   6.27e-07     6.67   0.000     2.95e-06    5.41e-06
bookd~yratio |   .0087405   .0032992     2.65   0.008     .0022684    .0152126
historical~5 |   .1135234   .0373304     3.04   0.002     .0402907    .1867562
    mkt_book |  -.0104156   .0036305    -2.87   0.004    -.0175377   -.0032935
       _cons |   .7475899   .0135611    55.13   0.000     .7209864    .7741934
```

***Source: LE Analysis***

## Figure 19:Evidence 6




***Source: LE Analysis***

As a final model, we added random effects (each company's beta has its own variance) and we also added a regressor on the standard deviation of company returns. The model now has overall circa 40% explanatory power. The interesting and relevant point is that there is still a statistically significant relationship between market cap and beta, suggesting that beta is systematically varying by market cap, and thus that beta might not be fully explaining excess returns for UK companies.

Naturally, an alternative way to test this is to regress excess returns from a portfolio of small companies on small company excess returns factors akin the Fama and French model. We have done this for a variety of portfolios using the Fama and French data, but we note that the company data from this section is for UK companies (whereas Fama and French data is from US returns and US listed companies).

**Figure 20:Evidence 7**

```
Random-effects GLS regression                   Number of obs      =      1260
Group variable (i): grpvar                      Number of groups   =         3

R-sq:  within  = 0.4214                         Obs per group: min =         8
       between = 0.0468                                        avg =     420.0
       overall = 0.3951                                        max =       953

Random effects u_i ~ Gaussian                   Wald chi2(5)       =    819.07
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
marketcapi~s |   3.50e-06   4.72e-07     7.41   0.000     2.57e-06    4.43e-06
bookd~yratio |   .0067977   .0026881     2.53   0.011     .0015292    .0120662
historical~5 |   .0679291   .0290004     2.34   0.019     .0110894    .1247688
    mkt_book |  -.0073849   .0029229    -2.53   0.012    -.0131136   -.0016561
standardde~e |   1.631803    .060398    27.02   0.000     1.513425    1.750181
       _cons |   .1414723   .0265736     5.32   0.000     .0893889    .1935556
-------------+----------------------------------------------------------------
     sigma_u |          0
     sigma_e |  .30408307
         rho |          0   (fraction of variance due to u_i)
------------------------------------------------------------------------------

.  xtreg  beta  marketcapinus  historicalgrowthinnetincomelast5  mkt_book

Random-effects GLS regression                   Number of obs      =      1351
Group variable (i): grpvar                      Number of groups   =         3

R-sq:  within  = 0.0338                         Obs per group: min =         8
       between = 0.0245                                        avg =     450.3
       overall = 0.0328                                        max =      1003

Random effects u_i ~ Gaussian                   Wald chi2(3)       =     45.74
corr(u_i, X)       = 0 (assumed)                Prob > chi2        =    0.0000
```

| beta | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| marketcapi~s | 4.01e-06 | 6.24e-07 | 6.41 | 0.000 | 2.78e-06 | 5.23e-06 |
| historical~5 | .092386 | .0368843 | 2.50 | 0.012 | .0200942 | .1646778 |
| mkt_book | -.0000881 | .0002097 | -0.42 | 0.674 | -.0004992 | .0003229 |
| _cons | .7377462 | .0126274 | 58.42 | 0.000 | .7129971 | .7624954 |
| sigma_u | 0 | | | | | |
| sigma_e | .41427083 | | | | | |
| rho | 0 | (fraction of variance due to u_i) | | | | |

***Source: LE Analysis***

Thus, the conclusion to this subsection is that the evidence for UK firms supports the hypothesis that CAPM betas for UK firms vary systematically by company size as measured by market cap, and this is independent of other factors. Thus the standard CAPM betas may be missing some explanatory power in terms of company risk premia and explaining company returns that are related to company size.

## 3.3 Estimation of the small firm equity premium

### 3.3.1 Small firm risk premium

In this sub-section, we estimate the value of the small firm equity premium using econometric analysis and the most up-to-date data available.

To estimate the value of the small firm premium, it is important to separate the premium into two separate segments; the portion of the premium that is subject to short-term fluctuations and the trend portion that is more sensitive to long-run movements. To conduct this estimation, statistics on US equity data from July 1927 to June 2010 was obtained from the Fama/French data library available online.

Using US equity data from NYSE, AMEX, and NASDAQ firms, benchmark portfolios are constructed based on size (market equity, ME) and book-to-market (the ratio of book equity to market equity, BE/ME). The size breakpoint (which determines the buy range for the Small and Big portfolios) is the median NYSE market equity. The BE/ME breakpoints are the 30th and 70th NYSE percentiles.

Benchmark factors are then constructed from the six size/book-to-market benchmark portfolios. The benchmark factors used are:

- The excess return on the market (Rm-Rf),
- The Small Firm Premium (SMB, Small Minus Big), measuring the performance of small stocks relative to big stocks, and
- The Value premium (HML, High Minus Low), measuring the performance of value stocks relative to growth stocks.

The excess return on the market (Rm-Rf) is the value-weighted return on all NYSE, AMEX, and NASDAQ stocks (from the CRSP database) minus the one-month Treasury bill rate.

The Small Firm Premium is the average return on three small portfolios minus the average return on three big portfolios.

–

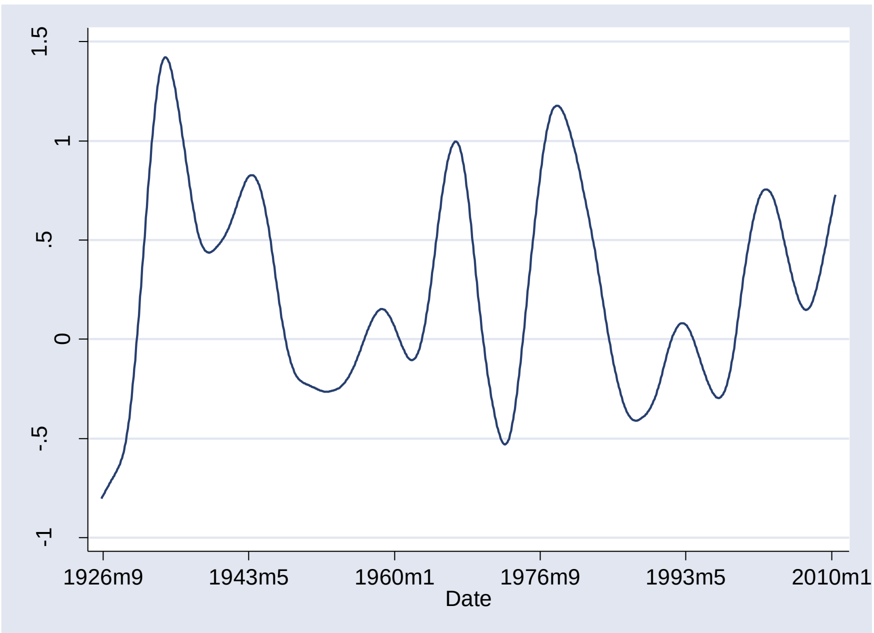
–

The Value Premium is the average return on two value portfolios minus the average return on two growth portfolios,

– –

In general, it is not intuitive or easy to pick a trend or average difference in the small company returns versus the large company returns, and the differences will be sensitive to the time periods chose. One method would be to choose the longest time period possible. Thus, in order to determine the value of the long run value component in the SMB series, a Hodrick-Prescott (HP) filter was applied to the SMB data series. The HP filter is a statistical technique used to obtain a smoothed representation of time-series data which is less sensitive to short-term fluctuations. The filter minimises a mathematical function, penalising departures in the series from the trend and changes in the rate of trend. A more detailed description of the HP filter is presented in Annex 3. The graphs of the HP filtered series are presented in Figure 3.21 and Figure 3.22.

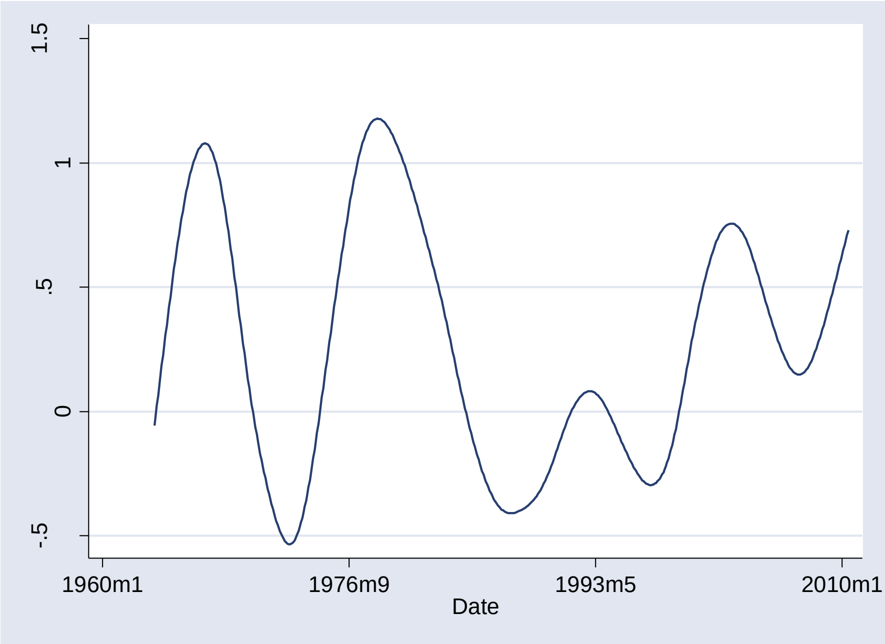
**Figure 3.21: HP Filtererd Trend SMB Values, 1926-2010**




***Source: LE analysis***

Across the full sample (July 1926 – June 2010) the filtered SMB series has an average monthly value of 0.242%, with a standard deviation of 0.507. Peaks occur in 1933, 1967 and 1978, with troughs in 1926, 1972 and 1988.

**Figure 3.22: HP Filtererd Trend SMB Values, 1963-2010**




***Source: LE analysis***

Across the recent sample (July 1963 – June 2010) average values for the filtered series are found to be 0.261 (a monthly increase of 0.019 over the full sample), with a standard deviation of 0.487). Peaks occur in 1966, 1978 and 2002, while troughs occur in 1972, 1987 and 1997. The peak to trough value of the series is 1.712.

Thus, the interpretation is that the most recent value of circa 0.242% (or 0.262%) for series starting in 1963) is the correct value of the difference in returns for returns of small companies over big companies (monthly). This works out to about 2.9% annually.

### 3.3.2 Small firm risk-factor beta

In addition to the small firm total risk premium, an additional question that should be addressed is that within the CAPM framework: What is the degree of “non-diversifiable” risk within the portfolio of the investor, or the company that should get a premium based on the small company SMB premium returns? There are two approaches to this.

One approach is to simply apply the small company premium data directly. An alternative is to estimate the SMB beta using regression techniques and then the result is the total factor times the total SMB premium.

Since we do not have company data because our companies are not listed, we must rely on some portfolio data from the Fama and French online data library. We note that our evidence on UK betas and academic research on the F&F 3-factor models supports the hypothesis that the small company premium should apply to the UK.

We present first regressions from the benchmark portfolios. These include small-high, small-medium, and small-low (split on size and value) portfolios. The key point is that some of the excess return in the portfolio is a non-diversifiable smallness premium.

For each of the portfolios, the relevant piece of data in the figure is the coefficient (beta) on the SMB factor (small minus big) and its p-value (P>|t|). The beta values are all close to one and the p-values close to zero, indicating statistical significance. It is also noteworthy that the R-squared statistics indicate the 3 factors explain circa 99% of total portfolio returns.

**Figure 3.23: Fama/French Three Factor Model Estimation Results**
**Three benchmark portfolios, July 1926- June 2010**

```
Small-high Portfolio
      Source |       SS       df       MS              Number of obs =    1008
-------------+------------------------------           F(  3,  1004) =48462.32
       Model |  71567.7142     3  23855.9047           Prob > F      =  0.0000
    Residual |  494.225749  1004  .492256723           R-squared     =  0.9931
-------------+------------------------------           Adj R-squared =  0.9931
       Total |    72061.94  1007  71.5610129           Root MSE      =  .70161

------------------------------------------------------------------------------
       sh_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.038146    .004432   234.24   0.000     1.029449    1.046843
         smb |   .9546522   .0072591   131.51   0.000     .9404076    .9688969
         hml |   .7800586   .0061116   127.64   0.000     .7680656    .7920516
       _cons |  -.0106972   .0223089    -0.48   0.632    -.0544747    .0330803
------------------------------------------------------------------------------


Small Medium Portfolio

      Source |       SS       df       MS              Number of obs =    1008
-------------+------------------------------           F(  3,  1004) =14610.98
       Model |  49913.2494     3  16637.7498           Prob > F      =  0.0000
    Residual |  1143.27009  1004  1.13871523           R-squared     =  0.9776
-------------+------------------------------           Adj R-squared =  0.9775
       Total |  51056.5194  1007  50.7016082           Root MSE      =  1.0671

------------------------------------------------------------------------------
       sm_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9792241   .0067408   145.27   0.000     .9659964    .9924517
         smb |   .8361912   .0110406    75.74   0.000     .8145259    .8578564
         hml |   .2873671   .0092954    30.92   0.000     .2691265    .3056077
       _cons |   .0334589   .0339305     0.99   0.324     -.033124    .1000418
------------------------------------------------------------------------------

Small Low Portfolio

      Source |       SS       df       MS              Number of obs =    1008
-------------+------------------------------           F(  3,  1004) =11713.25
       Model |  59628.7128     3  19876.2376           Prob > F      =  0.0000
    Residual |  1703.69014  1004  1.69690253           R-squared     =  0.9722
-------------+------------------------------           Adj R-squared =  0.9721
```

| Source | SS | df | MS | | |
|---|---|---|---|---|---|
| Total | 61332.4029 | 1007 | 60.9060605 | Root MSE = | 1.3027 |

| sl_rf | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| rm_rf | 1.10355 | .0082287 | 134.11 | 0.000 | 1.087402 | 1.119697 |
| smb | 1.061988 | .0134776 | 78.80 | 0.000 | 1.03554 | 1.088435 |
| hml | -.1888512 | .0113472 | -16.64 | 0.000 | -.2111181 | -.1665842 |
| _cons | -.1582482 | .0414201 | -3.82 | 0.000 | -.2395281 | -.0769682 |

*Source: LE analysis*

We also estimated the SMB beta using both the benchmark portfolio returns and using an industry-specific portfolio, where the industry is defined as "utilities" (the narrowest we could find out of the 49 available).

Testing the Fama/French Three Factor Model, NYSE, AMEX, and NASDAQ stocks were assigned to an industry portfolio across 49 industries. Portfolio returns were then calculated using equal equity weightings across each portfolio.

**Figure 3.24: Fama/French Three Factor Model Estimation Results Utilities Portfolio, July 1926- June 2010**

| Source | SS | df | MS | | |
|---|---|---|---|---|---|
| | | | | Number of obs = | 1002 |
| | | | | F( 3, 998) = | 586.47 |
| Model | 28930.0055 | 3 | 9643.33516 | Prob > F = | 0.0000 |
| Residual | 16410.0771 | 998 | 16.442963 | R-squared = | 0.6381 |
| | | | | Adj R-squared = | 0.6370 |
| Total | 45340.0825 | 1001 | 45.2947878 | Root MSE = | 4.055 |

| util_ew | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| rm_rf | .8404558 | .0256851 | 32.72 | 0.000 | .7900529 | .8908587 |
| smb | .1680783 | .041997 | 4.00 | 0.000 | .0856658 | .2504908 |
| hml | .4290438 | .0353484 | 12.14 | 0.000 | .359678 | .4984095 |
| _cons | .4196973 | .129324 | 3.25 | 0.001 | .1659192 | .6734753 |

*Source: LE analysis*

### 3.3.3 Net results—estimated small company premium (USA data)

We take the small-high growth company to be our benchmark estimate. The SMB beta is 0.95 and the monthly return on SMB from the filter is 0.242. Annualizing and multiplying this gives a total small company premium of 2.75%, or 275 basis points. We note also that the market beta is close to one, so there is no indication of a need to adjust the standard market CAPM beta, given the SMB beta.

Alternatively, we could use the estimation results from the Utilities Portfolio. These data are presented in Figure 3.24. We note however, that US utilities tend to be large firms, and the companies in the sample listed on the major US exchanges (NYSE, AMEX, NASDAQ, etc), and thus, the applicability of the utilities portfolio should be viewed as more limited, since we are not able to sort this into "small" utilities. In line with what would be expected for utilities firms, the coefficients on the excess return on the market (rm_rf), the small firm premium (smb) and the value premium (hml) are all positive and statistically significant. In conjunction with the average Trend SMB value, the model suggests that, across all equity returns in the Utilities portfolio, 0.49 percentage points of the annual cost of equity is accounted for by the Small Firm Premium. The model accounts for 63.8% of the variability in the data.

## 3.4 Estimation of Fama and French 3-factor model with UK data

In this section, we estimate the FFTM with UK data. One of the notable concerns UK regulators and the Competition Commission have had with the application of a Fama and French style model to the UK regulatory sphere has been the concern that the results were mostly based on US stock market data. Here we present evidence of the small firms' effect for the UK market.

The data have been provided by Professor Alan Gregory and are based on Gregory et. al. (2009). We use the data on monthly total returns for portfolios formed on size (market cap). We used a number of different definitions of the SMB factor, as well as the small company portfolio.

First, the selection of the time period is based on the longest time period available from the dataset. This is considered best practice, but we test the time period selection in the sensitivity analysis found in the annex and find our conclusions are not in general sensitive to the time period selected.

For our preferred model, we defined the small-minus-big[21] (SMB) factor as the average monthly return on the smallest 9 deciles (sorted by size, and market cap weighted returns) less the return on the largest decile. This makes the SMB factor related to firms that have at least well over £1bn market cap, according to the main market figures from LSE. The HMB, the market return, and the risk free rate of return are from Gregory et al (2009).

Further, we also defined the portfolio returns as the average return on the 9 smallest deciles of unweighted portfolios from the Gregory et al dataset[22]. The results of this regression are found in the figure below. The data used are the longest time period available from the Gregory et al (2009) dataset: q7:1980 to q12:2008.

[21] This is the monthly total return on the portfolio of small companies, less the return for big companies.

[22] Additional sensitivities are found in the annexes. In general, our conclusions were not sensitive to the formation of the portfolios or to the definition of the SMB factors.

**Figure 25:FFTM estimation results UK data**

```
reg  s1_9_rf  rm_rf smb hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 3367.53
       Model |  .859778215     3  .286592738           Prob > F      =  0.0000
    Residual |  .028510046   335  .000085105           R-squared     =  0.9679
-------------+------------------------------           Adj R-squared =  0.9676
       Total |  .888288261   338  .002628072           Root MSE      =  .00923


------------------------------------------------------------------------------
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9497979   .0109057    87.09   0.000     .9283455    .9712502
         smb |   .8457304   .0142919    59.18   0.000     .8176171    .8738436
         hml |  -.0211472   .0153934    -1.37   0.170    -.0514271    .0091327
       _cons |   .0007738   .0005102     1.52   0.130    -.0002298    .0017774
------------------------------------------------------------------------------
```

*Source: LE analysis*

The comparison of the above with the standard CAPM regression of the same portfolio on the market excess return factor shows a much better fit for the FFTM.

The coefficients on the market excess return factor (rm_rf) and the small-minus-big (SMB) factor are statistically significant at a high degree (P values close to zero/large t values). Likewise the range of the absolute values of the market excess return and smb is near to one. Conversely, the HML factor is not significant (although, since we did not use portfolios sorted on HML, this should not be surprising). Finally, the constant is not significant and is near zero--in general. Many 'tests' of the FFTM and the CAPM would use the constant close to zero as a criterion. However, our main concern is whether the SMB factor is significant and if the model fits the data better.

**Figure 26:CAPM regression and LR test results**

```
. reg  s1_9_rf  rm_rf

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  1,   337) =  570.22
       Model |  .558322217     1  .558322217           Prob > F      =  0.0000
    Residual |  .329966044   337  .000979128           R-squared     =  0.6285
-------------+------------------------------           Adj R-squared =  0.6274
       Total |  .888288261   338  .002628072           Root MSE      =  .03129

------------------------------------------------------------------------------
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .8776255   .0367525    23.88   0.000     .8053323    .9499186
       _cons |   .0031833   .0017067     1.87   0.063    -.0001738    .0065404
```

. lrtest s1_9_capm s1_9_ff_a1

likelihood-ratio test LR chi2(2) = 830.12

(Assumption: s1_9_capm nested in s1_9_ff_a1) Prob > chi2 = 0.0000

Note: Gregory et al (2009) data

***Source: LE analysis***

The appropriate statistical test of whether the better fit of the nested (the CAPM) model is a better versus the non-nested model (the FFTM) is the LR test, and the test statistic is given at the bottom of the figure. The FFTM is significantly better than the CAPM in terms of the LR test. Likewise, the Adjusted R-squared goes from 62.7% to 96.8%. Further, the constant is significantly bigger than zero, suggesting the 'alpha' of the portfolio is positive, or alternatively, that CAPM does not explain all the excess returns with the single market factor.

The implied small company premium is then the SMB coefficient estimate times the average return difference from the portfolio of small companies versus the portfolio of big companies (in this case the average of the value-weighted returns on the first (smallest size) 9 deciles less the largest ($10^{th}$) decile value-weighted return). The result of this exercise gives a premium of 263 basis points. It is notable that this result is very much in-line with the USA-based estimates (e.g., 275bps).

We also did sensitivity analysis using a variety of definitions of the SMB factor and the dependent variable portfolio returns. The different SMBs are based on different decile-portfolios with stocks sorted on size from the Gregory et al 2009 dataset (e.g., the smallest decile return less the largest decile return, etc.). We find that the small company premium is not sensitive to the SMB definition.

**Table 6: Sensitivity analysis small company premium-different SMB definitions**

| SMB definition on size deciles | 12 month return | beta smb | Small Company Premium |
|---|---|---|---|
| avg(10th-2nd)-minus-1st | 0.0313 | 0.84 | 0.0263 |
| avg(10th-3rd)-minus-avg(2nd-1st) | 0.0313 | 0.99 | 0.0310 |
| avg(10th-6th)-minus-avg(5th-1st) | 0.0313 | 0.92 | 0.0288 |
| 10th-minus-1st | 0.0313 | 0.96 | 0.0301 |
| SMB Gregory et al Definition | 0.0313 | 1.11 | 0.0348 |

*Source: LE*

## 3.5 Other evidence

We have undertaken private discussions with the AIGT companies with legacy assets. In general, they believe their own actual cost of equity and target cost of equity (for their parent company investors) to be much higher than the regulatory WACC. While these figures are confidential we can confirm that this is true; it might be useful, if Ofgem so desires, to engage with the companies on a one-on-one basis to discuss their information on a confidential basis.

We have also discussed the nature of a small company premium and the financial crisis with a number of equity analysts. There is universal agreement among them that there is a small cap premium and that the crisis has made it larger, rather than making it smaller. One analyst suggested that the impact of lay-offs from the crisis in the financial sector has meant that stock funds cannot cover as many funds as before, and therefore that smaller cap stocks are going to naturally get underweighted in the near terms (and so be potentially under-priced).

## 3.6 Conclusions

The best published research, practitioners' reports such as Duff and Phelps and Ibbotson, and our own empirical results on UK and US data all support the idea that small companies should receive an equity risk premium. This research suggests a small company premium of circa 250 bps in addition to the CAPM beta times the market equity risk premium.

There is little evidence to suggest that the UK is any different than the USA or other markets. Indeed, simply taking the difference between long run returns to small cap stocks in the UK yields significant premia for small cap stocks. The degree to which such premia are a) non-diversifiable and b) independent of the CAPM market risk premium must be estimated.

Based on our own regression models using the Fama and French three factor model, the premium is estimated to be 263bps.

There is a possibility that additional premia should be added for value/growth and illiquidity, but we have not investigated this in detail. The IGT gas companies are both small and in general high growth. It could be argued that there should be a growth or value premium to be added to the cost of equity as well.

The notion of an illiquidity premium has also been discussed and accepted in principle by other regulators. (Although the small cap premium itself may be capturing part of this).

There is significant regulatory precedent in the UK and elsewhere for including a small company equity premium. However, previously, regulators may have been reluctant to include small company premiums, but a large reason for this may have been that the companies in question perhaps were not that small, or the industry and regulatory regime was relatively stable, or that a standard CAPM seemed more reasonable (e.g., an estimated company beta was available). Alternatively, regulators also may have adjusted upwards some of the parameters in the standard or plain CAPM WACC, in order to implicitly adjust for smallness, illiquidity, or growth risks.

Nonetheless Ofgem had previously included a small company premium of 0.8% (80bps) in their 2002 decision for the legacy assets in the IGT sector. We believe that the recent financial crisis and the additional risk to the sector, and the evidence on the small cap premium presented warrant raising this to circa 263bps.

While we recognize some uncertainty in any estimation, we believe a reasonable range estimate for the small company risk premium would then be 80bps to 263bps, with the lower end based on the previous Ofgem estimate allowed and the higher end based on our own estimation, evidence, and judgment.

# 4 Cost of Debt

## 4.1 Existing Research on Small Company Debt Premium

### 4.1.1 Ratings agencies

One area of evidence that would be important to consider on the impact of company size on the company cost of debt is from the major ratings agencies, Standard and Poor's, Moody's and Fitch. Evidence that rating agencies for small businesses such as Dun and Bradstreet also indicates that size is a consideration.

The ratings agencies take company size into account when rating corporates. There are a variety of reasons for this, but one of the key reasons is that size enables the company to be more diversified and able to withstand shocks. Size may engender economies of scale and scope, giving competitive advantage as well.

It should be noted that in general, the regulatory cost of capital approach of Ofgem, and indeed most standard EU and North American regulators, is to assume that a company is able to maintain a given credit rating (typically A or BBB+, etc) in the context of the price control. The small companies in the current IGT review do not have credit ratings. As such, we are not aware that the previous cost of capital set for the legacy assets regulation had a target credit rating when setting the cost of debt in the cost of capital. Thus, while what we present in this subsection is specifically applied to the estimation of the credit ratings of a company, this will then directly impact the cost of debt.

In the Standard and Poor's full documentation of their corporate ratings methodology[23], S&P notes, "Standard & Poor's has no minimum size criterion for any given rating level. However, size usually provides a measure of diversification and often affects competitive issues. Obviously, the need to have a broad product line or a national marketing structure is a factor in many businesses and would be a rating consideration." "Still, small companies are, almost by definition, more concentrated in terms of product, number of customers, or geography. In effect, they lack some elements of diversification that can benefit larger firms. To the extent that markets and regional economies change, a broader scope of business affords protection. This consideration is balanced against the performance and prospects of a given business. In addition, lack of financial flexibility is usually an important negative factor in the case of very small firms. Adverse developments that would simply be a setback for firms with greater resources could spell the end for companies with limited access to funds. "

It should also be noted that there is a large body of research on explanatory variables that explain credit rates beyond credit ratings. For example, research has found that internationalization tends to lower borrowing costs. These variables are likely correlated with size.

### 4.1.2 Recent academic studies

This subsection reviews some of the literature on the relationship between company size and cost of debt. Generally, the findings of these articles agree that there is some small company debt premium, although not every paper supports this view.

[23] Standard and Poor 's (2010), « Rating Methodology: Evaluating the Issuer »

Frank and Goyal (2009) examine the relationship between firm size and leverage. The study uses a sample of US firms for the period 1950 to 2003 and measures firm size as the log of assets. A main finding is that firm size is an important factor in explaining a company's leverage and that bigger firms are able to sustain higher levels of leverage without degrading their overall cost of capital.

Chan et al. (2008) consider the relationships between a company's likelihood of default and a number of firm characteristics, including firm size. They find size is a factor in explaining default, and it is considered that credit spread is reflective of the expected cost of default. They reason that smaller firms have higher volatility of earnings/cash flows than larger firms and at the same time smaller firms usually have limited access to external finance compared to larger firms. Thus smaller firms are therefore more likely to default and hence should encounter higher costs of debt. The study empirically examines the relationship between company size and likelihood of default. Companies are divided into large cap (the largest 100 by market capitalisation), mid-cap (the next 200) and small-cap (the remaining companies). The analysis shows that small-cap firms are more likely to default. The firm default likelihood indicator estimated Chan et al. (2008) is around 2 percentage points higher for small-caps than for big caps for 2003. In addition, the likelihood of default of small firms is more sensitive to business cycle than for larger firms.

Campello et al. (2007) construct the size and book-to-market factors using the Fama and French (1993) two-by-three sorting in size and book-to-market methodology. Using a sample of bonds issued by 1205 non-financial firms between January 1973 and March 1998, their empirical analysis found expected risk premia for size and book-to-market of 3.47% and 1.91%, respectively. Thus they conclude that the evidence is consistent with the argument that book-to-market and size capture important aspects of risk that are expected to be priced in returns.

An article by Bhojraj and Sengupta (2003) examined the impact of a number of company characteristics on the spread of corporate bond yields over the US Treasury bond rate and also on companies' overall credit ratings. In this study, company size was defined as a firm's total assets at the end of the reference year.

The relationships were estimated using data on bonds issued during the period 1991 to 1996, with just over a thousand usable observations. Bhojraj and Sengupta (2003) use four different model specifications, including different combinations of corporate governance indicators. In each of these models the company size coefficients are statistically significant, and indicate that smaller companies face higher yields and have lower credit ratings.

In a study by Minard, Sanvincente, and Artes (2008)[24], the authors collected corporate credit rating grades data from Standard & Poor's and Moody's and accounting information for a sample of 627 American companies. They then ran a stepwise ordered logit regression model to select the variables that better explain agency credit ratings. Their results found that the most important variables in determining credit ratings are size, financial leverage, operating performance and volatility.

In one of the most recent studies available from an ECB working paper (Petrasek 2010)[25], the author considers determinants of the cost of debt and in particular multimarket trading. In this

[24] Working Paper available at: http://www.crc.man.ed.ac.uk/conference/archive/2007/papers/minardi-andrea-estimating-credit-rating.pdf.

[25] Petrasek, Lubomir (2010) ECB WORKING PAPER SERIES NO 1212 / JUNE 2010 MULTIMARKET TRADING AND THE COST OF DEBT EVIDENCE FROM GLOBAL BONDS1.

study, the author finds that globally traded bonds trade at a premium (lower yield) to domestic bonds, even for the same companies. The premium is about 25bps. Naturally, the implication is that companies that can access the global credit markets will have a lower cost of debt than domestic companies.

## 4.2 Small Company Cost of Debt

This section makes an empirical analysis of the cost of debt for small companies using the spread to government bonds of bonds issued by UK companies. The aim is to determine whether and to what extent small companies pay higher interest on their debt compared with larger companies.

The approach taken to measure the size of a company is to sum the company's total debt (bonds and loans) and the company's market capitalisation. Small companies are then defined as the smallest 10 per cent of companies according to this measure.

### 4.2.1 Data

Data on spreads to government bonds for all bonds issued in the UK with maturity dates before 1st January 2050 were collected from Bloomberg. For each bond, the market capitalisation and total debt of the issuer are also included in the dataset. In addition the date of maturity of each bond is included in the dataset.

### 4.2.2 Model specification

The left hand side variable is the spread of each bond to an equivalent government bond. The main explanatory variable of interest is the value of the company, given by the sum of the issuing company's debt and market capitalisation. In addition to this, the model also includes the term to maturity for each bond, given by the number of days until maturity.

The equation estimated is:

$$Spread_{ij} = \alpha + \beta_1.log(Term\ to\ maturity_{ij}) + \beta_2.log(Total\ company\ value_{ij})$$

Where: $Spread_{ij}$ is the spread between the rate on bond 'i' issued by company 'j' and the rate on an equivalent government bond;

*Term to maturity* is the number of days to maturity for bond 'i' issued by company 'j';

*Total company value* is the sum of the total debt (bonds and loans) and market capitalisation of company 'j'.

Descriptive statistics of the variables included in the model are presented in Table 7 below.

Since many companies have multiple bonds outstanding (all of which are included in the dataset) either a fixed effects or random effects approach can be used in order to control company specific factors, such as a company's credit rating.

The Hausman test is used to determine the whether a fixed effects or random effects approach is most appropriate (the results of the test are presented in the technical annex). The results of the test (a large p-value of 0.39) indicate the random effects model should be used.

**Table 7: Descriptive statistics of the variables used**

| Variable | Number of observations | Mean | Standard deviation |
|---|---|---|---|
| Spread | 2574 | 217.31 | 335.89 |
| Log term to maturity | 17695 | 6.51 | 1.50 |
| Log total company value | 11965 | 25.09 | 0.87 |

***Source: LE Analysis***

### 4.2.3 Regression results

The regression results display a statistically significant relationship between spread and the total value of the company issuing a bond (Table 8). This indicates that the cost of debt is higher for small companies compared with companies in general. The actual and predicted spreads from the model are plotted together against the log of total company value in Figure 27.[26]

**Table 8: Regression results (random effects estimation of equation: $Spread_{ij} = \alpha + \beta_1.log(Term\ to\ maturity_{ij}) + \beta_2.log(Total\ company\ value_{ij})$)**

| Dependent variable: Spread | | Coefficient | Standard error |
|---|---|---|---|
| Log term to maturity | | 36.77*** | 3.49 |
| Log total company value | | -47.88*** | 13.04 |
| Constant | | 1087.46*** | 302.81 |
| R squared: | Within | 0.1055 | |
| | Between | 0.1150 | |
| | Overall | 0.1588 | |

Note: "***" signifies that the coefficient is statistically significant at the 1% level.
***Source: LE Analysis***

The sensitivity of these results is examined by including the log of market capitalisation instead of the log of total company value. The results of this alternative method differ only slightly from the results of the original regression (Table 9).

**Table 9: Regression results (random effects estimation of equation: $Spread_{ij} = \alpha + \beta_1.log(Term\ to\ maturity_{ij}) + \beta_2.log(Market\ capitalisation_{ij})$)**

| Dependent variable: Spread | | Coefficient | Standard error |
|---|---|---|---|
| Log term to maturity | | 36.77 | 3.49 |
| Log market cap | | -47.94 | 13.04 |
| Constant | | 1088.87 | 302.77 |
| R squared: | Within | 0.1055 | |
| | Between | 0.1153 | |
| | Overall | 0.1589 | |

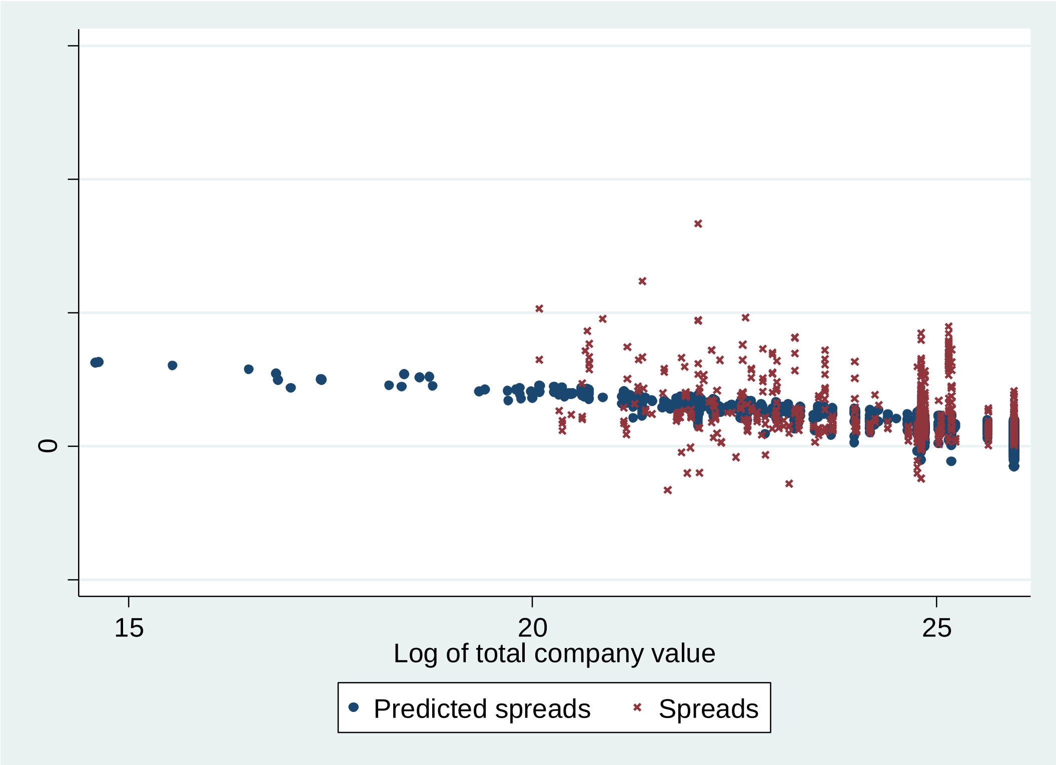
***Source: LE Analysis***

In order to examine the extent that the cost of debt is higher for small companies we define a company as "small" if it is in the smallest 10 per cent of companies in terms of value (total debt + market capitalisation).

[26] Plots of the residuals are presented in the technical annex.

Inputting average company value and average term to maturity for small companies into the estimated model shows that the spread for an average small company is 471 basis points. This compares to 223 basis points for other companies. Hence, the spread premium for small companies predicted by the model is 248 basis points.

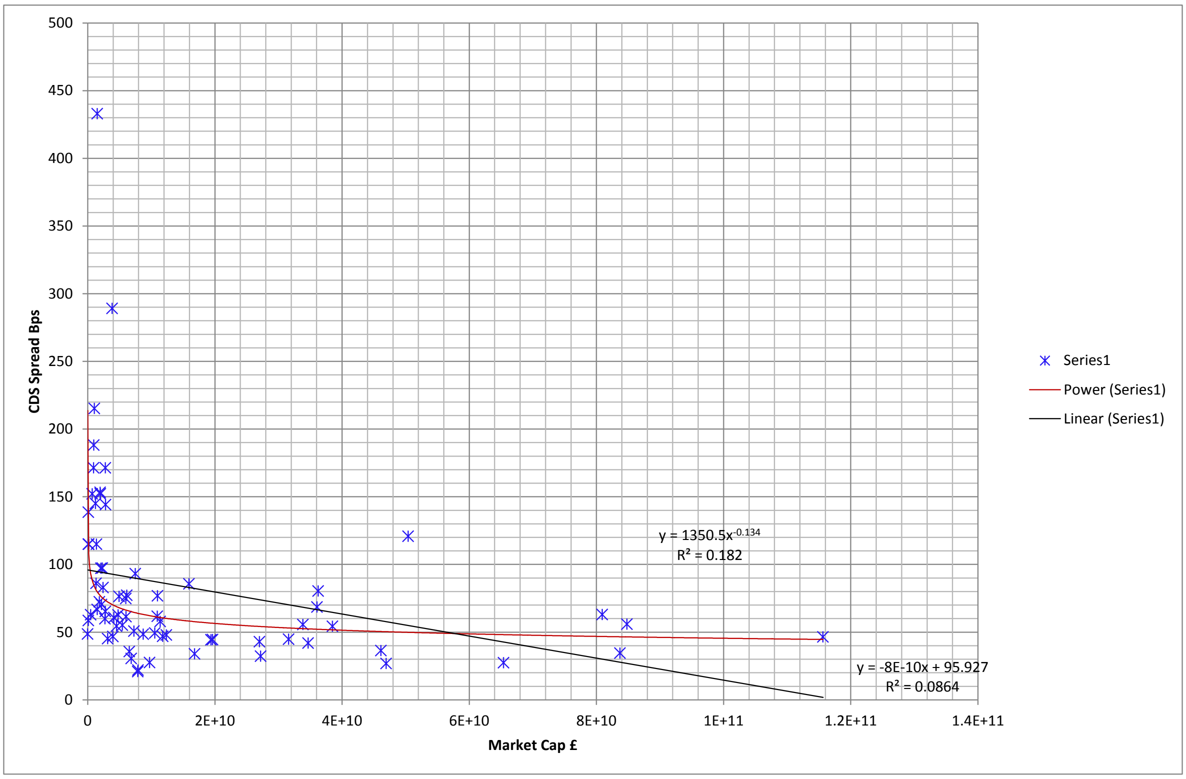
**Figure 27: Plot of predicted and actual spreads vs. log of total company value**




***Source: LE Analysis***

As an additional check and analysis of our modelling of the cost of debt and bond yields as a function of company size, we also considered the measure of the default risk premium using data from credit default swaps. These are derivative contracts that are effectively insurance on the default, and Bloomberg reports the implied credit spread (CDS spread) for each security. The dataset included European CDS and approximately 70 companies had usable data on both company market cap, company total debt value, and a CDS spread. In general, these will be larger companies that have CDS spreads, as companies that are not listed or do not have listed debt will not have CDS's that trade on their debt. Nonetheless we find an apparently significant impact of company size, as measured by market cap in £ on the spreads. The estimated impacts are found in the figure below. While there is some sensitivity to whether a linear or power-model (the power model is akin to a constant elasticity model) is chosen, the power model is the better fitting model. The implication of the power model is that for very small companies the cost of debt will increase quite rapidly at a certain small size.

**Figure 28:Company size and CDS implied spreads**




***Source: LE analysis of Bloomberg data***

## 4.3 AIGT company evidence

A number of the AIGT companies have provided us with confidential information regarding their current borrowing costs, and likely near terms costs of refinancing. Since these companies actively compete for new customers, new and near term borrowing rates are considered very sensitive commercial information. However, companies agreed that London Economics could reveal an indicative range of borrowing rates. In general, rates are given in terms of spreads over benchmark rates, such as Sterling LIBOR. The rates we have seen have been on the order of 250 to 400 bps. Our conclusion is that the size premium of circa 270 to 360bps exists in general for these companies.

One of the AIGT companies with legacy assets has had its refinancing recently made public and the details of this are as per below. The premiums are, we would argue, right in-line with our current estimates. We note that the two and a half year term of the refinancing is a risk element that could be significant.

"17 December 2009; Inexus has successfully refinanced its £461 million debt facilities. The existing bank group has extended the terms of the £461 million debt facilities to 31 August 2012…. The senior debt facilities have been priced at an initial margin of 250 basis points and the junior debt facility has been priced at an initial margin of 450 basis points. Both the £426 million senior debt facilities and the £35 million junior debt facilities will mature in August 2012."

Fees on debt also form a significant portion of debt financing costs for the AIGT companies.

## 4.4 Other factors influencing cost of debt

### 4.4.1 Availability

Availability of finance is another factor which influences the cost of debt. In general, there is a large body of research that suggests small firms could be financially constrained (cash or liquidity constrained); in other words, their capital and investment cash flow needs are not always met by the financial markets. The hypothesis for why this is so tends to focus on market information, costly information (it being more costly per unit to uncover information on small firms), and informational asymmetry, all of which are likely to be more of an issue for small firms.

We have not undertaken a detailed study of the liquidity needs and availability of cash for the AIGT firms with legacy assets in question. We have, however, discussed the availability of finance and cash with the firms' managerial teams. In general, we would argue that, based on these conversations, the availability of debt finance for the AIGT companies and a small company in general is an issue.

While we have not undertaken a rigorous study of cash-flow constraints of the AIGT firms, we argue that a simple and intuitive evidence of their positions has to do with the terms and maturity of the debt finance these companies receive. In general, debt finance has been done via syndicated bank loans, and not via bond markets. The maturities are generally short (e.g., in the 2-5 years range). In general, it is by far the most common corporate finance practice in our experience (and seemingly common risk-management sense) to match maturities to asset lives where possible (in other words, sell a 20-year bond for a 20-year lived asset financing), and thus avoid the risks associated with needing to continually refinance.

### 4.4.2 Fees

We have been made privy to terms and conditions of loans and financing for some of the AIGT companies. While the firms in question have asked us to keep all such terms confidential, we would note that in general, fees and terms for smaller and syndicated loans tend to be more expensive on a per unit basis that for larger debt financing, and that since there tends to be economies of scale in debt financing, that this should be intuitive. In other words, the due-diligence of the bankers, the arrangement efforts, per unit of borrowing, should tend to fall as the size of the loan/debt financing increases.

The debt spread we use includes a small company fees/costs adder of 10bps on the cost of debt, as there are fixed elements of fees for raising debt. We would note that in fact, total fees for some IGT companies that are actually incurred are likely to be higher. Evidence from the IGT members confirms the experience of smaller companies/smaller loan amounts incurring arrangement and commitment fees of circa 26bps. While we have seen evidence from the various AIGT companies that fees could be significantly higher, because of the confidential nature of these, and also because of the difficulty in converting fees, such as non-utilisation fees[27], into a basis points/percentage points adder, we have not included these.[28]

[27] It would be very difficult to estimate, what percentage of the loan facility is expected to be drawn down over the period; this would then have to be spread over the period as an expected value, and converted to a percentage by the total (uncertain) drawdown, for example.

[28] We have advised the AIGT companies to consider liaising with Ofgem directly on these fees and evidence thereof.

It is notable that in a previous regulatory decision, the CC allowed an adder for fees on the cost of debt.

### 4.4.3 Risk premia and diversifiability in the cost of debt

An additional point that is sometimes raised when considering the cost of debt is whether the risk premium as a default spread should be included in the regulatory cost of debt—we argue it should. Eliminating a diversifiable debt premium is perhaps only done in practice when evaluating fund managers that include risky debt in their portfolios, whereas regulators, to the best of our knowledge, universally use the cost of debt directly. The reason for this is perhaps quite simple; the cost of debt is in most cases directly observable. So while it is sometimes argued that the regulatory cost of debt could be lower or higher, based on target or achievable gearing, credit ratings, etc., there is still an observable cost of debt for almost every company.

### 4.4.4 Assets/Collateral

In general, utilities that are regulated tend to have somewhat lower costs of debt that the typical industrial company. However, for small companies such as the AIGT companies, one factor which is notable which would tend to moot such an effect; the IGT firms are not able to use their grid assets as collateral due to regulation.

## 4.5 Conclusions regarding the cost of debt

Overall, the available evidence, our own models, and previous work predicts a size premium for debt of about 250bps (the total cost of debt for small companies had a circa 450bps premium). This was very much in line with recent IGT sector financing (250bps for senior debt and 450bps for junior). There is regulatory precedent for a 10bps premium for fees and other costs as well (CC review of CAA Stansted Decision).

The evidence and regulatory precedent for an average regulated company cost of debt premium is in the range of 100 to 250bps. (Bristol Water appeal, Ofwat used 160bps.) Ofgem previously allowed a cost of debt premium for the IGT sector of 200 to 300bps. Ofgem does not explicitly state that there is a small company debt premium, but the debt premium is evidently larger than some other regulatory decisions. To convert the 2002 Ofgem debt premium into a small company debt premium, it is necessary to consider the change in the premium over time, and to consider how much larger the 2002 premium was over standard regulated utilities debt premium at the time. One way to do this is to compare the IGT premium with a similar allowed Ofgem debt premium. Ofgem’s 2004 Distribution price control allowed a debt premium range of 1.0 to 1.8% (100 to 180bps). Taking the 2004 DSO price control review as the closest regulatory decision to the IGT 2002 decision, and taking the midpoint of this range yields an implied debt premium of circa 100bps (200 less 100=100 or 300 less 180=120, that is subtracting the low range from the low end and high end from the high end of the ranges). This implies a previous implicit small company premium of 100 to 120bps for the IGT sector. This previous range was, however, at a time of financial security and finance boom, with availability of short term cash finance by banks much greater pre-crisis, than as of post. Thus, using the implied small company premium from the 2002 Decision (100 to 120bps) and applying that to the recent 160bps from Ofwat, would give a total debt premium of 260bps to 280bps.

We take this 260bps value as the lower end of the range. Our model predicted value of 450bps, and the actual company evidence was 250 and 450bps. But given that companies will have some senior and junior debt, and a mix of debt that rolls over given any particular time[29], we believe a slightly lower figure is justified. Given that the only public evidence on the cost of debt to the IGT companies suggests a 250 to 450 range between junior and senior debt, and the midpoint of this range is 350bps, we take this to be the upper end of the range.

Allowing an additional 10bps for fees, we therefore argue that the overall debt premium for the IGT sector should be in the range of 270 to 360bps.

[29] Although, we have been informed by the AIGT companies that some of these companies will have to roll virtually all of their debt at the same time. The exact timing of these items is considered confidential by the companies, but Ofgem and the companies may want to liaise directly on this point.

# 5 WACC Estimates for IGT Companies and Conclusions

## 5.1 WACC

In this section we present LE's WACC estimates for the IGT companies. We note from our previous discussion that the focus of our estimates is on the debt and the equity premia for the small companies in the IGT sector.

We provide a discussion of each of the WACC main parameters following on from the table below. The table is colour coded, in that the new parameters which are derived from LE estimates are in green, the figures which are taken directly from previous decisions are in blue, and the derived figures are in black.

| | Ofgem IGT | | DPCR5 | CC re Bristol Water | | LE 2010 Estimates | |
|---|---|---|---|---|---|---|---|
| | Feb-02 | | Dec-09 | Jun-2010 | | Oct-2010 | |
| | Low | High | Final | Low | High | Low | High |
| **Cost of debt:** | | | | | | | |
| Risk free rate | 2.75 | 2.75 | 2 | 1 | 2 | 1.5 | 1.5 |
| Debt risk premium | 2 | 3 | 1.6 | 2 | 3 | 2.7 | 3.6 |
| Cost of debt | 4.75 | 5.75 | 3.6 | 3.9 | 3.9 | 4.2 | 5.1 |
| **Cost of equity:** | | | | | | | |
| Risk free rate | 2.75 | 2.75 | 2 | 1 | 2 | 1.5 | 1.5 |
| Equity risk premium for the market | 3.5 | 3.5 | 5.25 | 4 | 5 | 4.5 | 4.5 |
| Gearing | 37.5% | 37.5% | 65% | 60% | 60% | 37.5% | 37.5% |
| Equity Beta | 0.7 | 1 | 0.9 | 0.64 | 0.92 | 0.85 | 0.85 |
| Small company premium | 0 | 0.8 | 0 | 0 | 0 | 0.8 | 2.63 |
| Post tax cost of equity | 5.2 | 7.05 | 6.73 | 3.56 | 6.6 | | |
| Taxation adjustment (multiplier) | 1.43 | 1.43 | 1.39 | 1.39 | 1.39 | 1.39 | 1.39 |
| Pre-tax cost of equity | 7.4 | 10.1 | 9.3 | 4.9 | 9.2 | | |
| **Real pre-tax WACC** | **6.4** | **8.5** | **5.6** | **4.3** | **6** | **6.90** | **8.82** |
| **Vanilla WACC** | **5.03** | **6.56** | **4.69** | **3.76** | **4.98** | | |

***Source: LE and Ofgem***

### Risk free rate (rfr):

We have not studied the issue of the risk-free rate in any detail. As the risk free rate tends to be something that has changed apparently due to lower interest rates then it is best to take the most recently available rfr. The range given by the CC in their most recent findings regarding the appeal of Ofwat's decision by Bristol Water is the basis for our calculation. We note that this is a lower rate at the high and low ends of the range than Ofgem's estimates in 2002. The value used is 1.5%.

### Debt risk premium

We have presented our detailed case in the section on the small company debt premium, and the conclusions found there are the bases of our estimates. We estimate a range of 270 to 360 basis points.

**Cost of debt**

The overall cost of debt is then the sum of the premium and the risk free rate.

**Cost of equity**

The cost of equity is the risk free rate, plus any equity risk premia that are not diversifiable. The risk factors are a) the market, b) the small company premium and c) the growth/value premium. We have included a market premium along the lines of the Sharpe-Lintner CAPM (standard CAPM) and a small company premium using a UK-data-based Fama and French 3-factor model.

*Market beta and market equity risk premium*

The market beta estimates we use come from Ofgem's previous 2002 results; we use the mid-point of Ofgem's range (0.85). (We note that the market betas in our own 3-factor model regressions had coefficients close to one, but it is our judgment that regardless of methodology, the correct value should most likely be close to 1. We note that the previous Ofgem IGT decision used a value of 1.)

Further, the cyclical risk of the IGT sector relative to the market has likely increased since 2002 (indicating a possibly higher beta). The cyclicality of the sector is important as it indicates possible non-diversifiable risk (and not captured in standard utility proxy betas). The cyclicality is due to the strong positive correlation between new connections and new construction. Also, unlike standard regulated utilities, the AIGT companies do not have the ability to pass-on variations in opex and other on-going spend, either via the formula (e.g., indexed to cost-profiles, RPI, etc.), or upon review every five years and resetting the base price/allowed cost.

Nonetheless we've used the midpoint of the values 0.7 to 1.0 from Ofgem's 2002 IGT decision. We used the mid-point of these values for our estimates, 0.85. We would highlight that our beta is lower than the previous beta used by Ofgem, but rather than adjusting beta upwards we estimate a small company premium directly. We note that the standard CAPM generally works well for standard utilities, but for small companies, it is our judgement that the standard CAPM does not work well.

The market risk premium comes from the Bristol water appeal, mid-point of the range (4 to 5) of the CC decision, 4.5%.

*Small company premium*

As we have taken mid-point values from the previous regulatory decisions, we have focused on the small company premiums for equity and debt.

Our estimates of the small company equity premium are based on:

1) The previous Ofgem IGT decision of 2002, and the fact that the IGT market has become more risky, and that financing for small firms has become more challenging.

2) Long-term estimates of the small company premium based on company returns in the UK and elsewhere.

3) Academic evidence.

4) Our own regression models of the cost of equity and the size premium.

5) Smallness may proxy for illiquidity, growth/value, market inefficiency, and variety of factors. The practical notion and professional judgment is that small firms are more risky, and that this additional risk is not diversifiable, and should therefore earn a premium rate of return. We have not double counted the possible additional premia that may exist.

We estimate the small company equity premium to be 80 bps to 263bps.

**Pre-tax WACC**

We have focussed on the pre-tax cost of capital. Our estimates of the WACC are in the range from 6.90% to 8.82%.

## 5.2 Conclusions

In response to Ofgem's consultation letter, we have estimated the cost of capital for the IGT sector legacy assets.

We have focussed on the pre-tax cost of capital. Our estimates of the WACC are in the range from 6.90% to 8.82%.

The IGT sector is fundamentally different from other sectors and/or companies that Ofgem regulates because a) it is competitive, b) it is very small, c) the price control for legacy assets is purely based on the cost of capital c) does not have pass-through on non-controllable costs d) is more cyclical, and e) has certain regulatory constraints and differences which are more risky than GDNs (e.g. metering).

Given the above, and also given recent decisions of Ofgem, other UK regulators (e.g., Ofwat) and findings of the Competition Commission (CC) from the Bristol Water appeal, we have only focused on estimating a small company premium for the cost of debt and the cost of equity. For all of the other WACC parameters we have relied on previous estimates. For the gearing and beta assumptions, we relied on the previous Ofgem IGT sector WACC; for the risk free rate, and the market risk premium (which are more time-sensitive), we relied on the recent CC findings in the Bristol Water Ofwat appeal.

For the small company debt premium, we provide LE's own estimates. The estimates are based on a) the actual data on recent borrowings in the sector b) previous regulatory findings—and the fact that the financial crisis has most likely raised the cost of debt relative to the previous 2002 Ofgem Decision, c) our own modelling of debt spreads using two different approaches (regression using debt spreads and UK bonds, and using CDS spreads), and d) our own professional judgment. The sum of this evidence suggested a debt spread in the range of 270 to 360 basis points over the risk free rate.

The debt spread we use includes a small company fees/costs adder of 10bps on the cost of debt, as there are fixed elements of fees for raising debt. We would note that in fact, total fees for some IGT companies that are actually incurred are likely to be higher. Evidence from the IGT members confirms the experience of smaller companies/smaller loan amounts incurring additional arrangement and commitment fees of circa 26bps. While we have seen evidence from the various AIGT companies that fees could be significantly higher than 10bps, because of the confidential nature of these, and also because of the difficulty in converting fees, such as non-utilisation fees[30], into a basis points/percentage point adder, we have not included these.[31]

For the cost of equity, we also have provided our own estimates. The estimates are based on a) existing international, professional, and academic research on the small company premium, b) previous regulatory findings, c) our own modelling of the cost of equity small company premium using two different approaches (regressions using the Fama and French 3-factor model; regressions on UK data using the Fama and French methodology. We also present data on UK company betas and company size classes), and d) our own professional judgment.

Overall, we suggest that the upper end of the range chosen is the correct one for Ofgem to choose. This is because a) the IGT firms face risks that are higher than normal regulated firms b) the overall approach to regulation of the legacy assets of IGT firms is of a reasonable profits test based on WACC, rather than a standard full price control, and c) the costs/prices of the IGT sector have already satisfied an additional hurdle rate given that these are companies that are fully competitive when bidding for new sites d) Ofgem previously chose the upper end of the range, e) the financial crisis means that risks relative to 2002 levels are now much higher.

[30] It would be very difficult to estimate, what percentage of the loan facility is expected to be drawn down over the period; this would then have to be spread over the period as an expected value, and converted to a percentage by the total (uncertain) drawdown, for example.

[31] We have advised the AIGT companies to consider liaising with Ofgem directly on these fees and evidence thereof.

# 6 References


Banerjee, A. and Duflo, E. (2008). “Do Firms Want to Borrow More? Testing Credit Constraints Using a Directed Lending Program”, Mimeo, MIT.

Banz, Rolf W. (1981), “The relationship between return and market value of common stocks”, March, Journal of Financial Economics.

Barad, M.W. (2003), “Ibbotson Style Indices: A Comprehensive Set of Growth and Value Data”, Ibbotson Associates.

Beck, T. & Demirguc-Kunt, A. (2006). "Small and medium-size enterprises: Access to finance as a growth constraint", Journal of Banking & Finance, 30(11), 2931-2943.

Berger, A.N., Udell, G.F., 1998. "The economics of small business finance: The roles of private equity and debt markets in the financial growth cycle." *Journal of Banking and Finance,* 22, 613–673.

Bhojraj, S. and Sengupta, P. (2003). *Effect of Corporate Governance on Bond Ratings and Yields: The Role of Institutional Investors and Outside Directors*. Journal of Business, vol. 76, issue 3, pages 455-476.

Campello, M., Chen, L. and Zhang, L. (2007), *Expected Returns, Yield Spreads, and Asset Pricing Tests*. Review of Financial Studies, Forthcoming.

Chan, H., Faff, R. W., and Kofman, P. (2008) *Default Risk, Size and the Business Cycle: Three Decades of Australian Asset Pricing Evidence*. Working Paper, available at SSRN: http://ssrn.com/abstract=1097444

Cuthbertson, Keith ,Dirk Nitzsche, Niall O'Sullivan (2008), “UK mutual fund performance: Skill or luck?”, Journal of Empirical Finance 15 (2008) 613–634.

Damadoran, A. (2001), *Investment Valuation*, New York: Wiley.

De Mel, S., McKenzie, D. and Woodruff, C. (2008). "Returns to Capital in Microenterprises: Evidence from a Field Experiment", *Quarterly Journal of Economics*, 123(4), 1329-1372.

Dimson, E., Nagel, S. and Quigley, G. (2003) “Capturing the Value Premium in the U.K. 1955-2001”, Financial Analysts Journal

Fama, E.F. and French, K.R. (2007); “The Value Premium and the CAPM”, Journal of Financial Economics, Elsevier

Fama, E., and French, K. (2007), “Dissecting Anomalies”, CRSP Working paper #610.

Fama, E, and French,K., (1997), “Value versus Growth: The international Evidence” Tuck Business School working paper.

Fama, E., French, K., (1993) “Common risk factors in the returns on stocks and bonds”, Journal of Financial Economics.

Frank, Murray Z., and Vidhan K. Goyal, (2009). *Capital Structure Decisions: Which Factors are Reliably Important?* Financial Management, 38, 1-37.

Giannetti, M. (2001). *Risk Sharing and Firm Size: Theory and International Evidence*. Working Paper, available at SSRN: http://ssrn.com/abstract=262788 or doi:10.2139/ssrn.262788.

Grabowski, R., ”Risk Premium Report 2010” - Duff and Phelps LLC, 2010

Gregory, Alan, Rajesh Tharyan and Angela Huang Xfi (2009), “The Fama-French and Momentum Portfolios and Factors in the UK”, Centre for Finance and Investment, University of Exeter Paper No 09/05 December 2009.

Heston, Steven L., Rouwenhorst, K. Geert, and Wessels, Roberto E. (1995), “The structure of international stock returns and the integration of capital markets”, Journal of Empirical Finance 2, 173- 197.

Jaffee, D. and T. Russell (1976). “Imperfect Information, Uncertainty and Credit Rationing,” *Quarterly Journal of Economics,* 90, 651–666.

Jun, D. and Malkiel, B.G. (2008), ”New Paradigms in Stock Market Indexing”, European Financial Management, Vol. 14, Issue 1, pp. 118-126, January.

Kilbert, M. and Subramanian, R.A. (2010), “Revisiting Global Small Caps”, MSCI Index Research Group.

London Economics (2002), "Study on the "quantification of the macro-economic impact of integration of EU financial markets", http://ec.europa.eu/internal_market/securities/overview_en.htm

Petrasek, L. (2010), “*Multimarket Trading and the Cost of Debt Evidence from Global Bonds*”, ECB Working Paper Serices No. 1212/June.

Piketty, T. (1997), “The Dynamics of the Wealth Distribution and the Interest Rate with Credit Rationing,” *Review of Economic Studies*, 64, 173–189.

Standard and Poor’s (2010), “Rating Methodology: Evaluating the Issuer.”

Stiglitz, J. and A. Weiss (1981). “Credit Rationing in Markets with Imperfect Information,” *American Economic Review*, 71, 393–410.

Van Binsbergen, Jules H., Graham, John R. and Yang, Jie, (2010). *The Cost of Debt*. Journal of Finance, Forthcoming; WFA 2007 Big Sky, MT Meetings Paper; EFA 2007 Ljubljana Meetings Paper.

# Annex 1 Details of Sensitivity Analysis of UK FFTM

## A1.1 Sensitivities cases

### A1.1.1 Check of sensitivity of the models to the time period selected

In order to check the sensitivity of the FFTM to the period of time selected, we simply used our preferred models and tested whether dropping some decades would change the results. We chose to drop the first few years (1980-89), some middle (the 1990s), and years >2000. The results are found below. We conclude the model is not particularly sensitive to the time period selected.

```
. reg  s1_9_rf  rm_rf smb_a1 hml if year>1989

      Source |       SS       df       MS              Number of obs =     228
-------------+------------------------------           F(  3,   224) = 2071.83
       Model |  .554491006     3  .184830335           Prob > F      =  0.0000
    Residual |  .019983302   224  .000089211           R-squared     =  0.9652
-------------+------------------------------           Adj R-squared =  0.9647
       Total |  .574474308   227  .002530724           Root MSE      =  .00945

------------------------------------------------------------------------------
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9198651   .0149286    61.62   0.000     .8904466    .9492836
      smb_a1 |   .8335145   .0166379    50.10   0.000     .8007277    .8663013
         hml |  -.0288262   .0177537    -1.62   0.106    -.0638118    .0061593
       _cons |   .0006457   .0006294     1.03   0.306    -.0005946    .0018861
------------------------------------------------------------------------------


. reg  s1_9_rf  rm_rf smb_a1 hml if year<2001

      Source |       SS       df       MS              Number of obs =     243
-------------+------------------------------           F(  3,   239) = 2802.50
       Model |  .568579403     3  .189526468           Prob > F      =  0.0000
    Residual |  .016163034   239  .000067628           R-squared     =  0.9724
-------------+------------------------------           Adj R-squared =  0.9720
       Total |  .584742437   242  .002416291           Root MSE      =  .00822

------------------------------------------------------------------------------
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9595805   .0114925    83.50   0.000     .9369409      .98222
      smb_a1 |   .8915094   .0158733    56.16   0.000       .86024    .9227788
         hml |   .0106558   .0159883     0.67   0.506    -.0208402    .0421518
       _cons |   .0003794   .0005401     0.70   0.483    -.0006846    .0014434
------------------------------------------------------------------------------


. reg  s1_9_rf  rm_rf smb_a1 hml if year>1989

      Source |       SS       df       MS              Number of obs =     228
-------------+------------------------------           F(  3,   224) = 2071.83
       Model |  .554491006     3  .184830335           Prob > F      =  0.0000
    Residual |  .019983302   224  .000089211           R-squared     =  0.9652
-------------+------------------------------           Adj R-squared =  0.9647
       Total |  .574474308   227  .002530724           Root MSE      =  .00945
```

```
------------------------------------------------------------------------------
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9198651   .0149286    61.62   0.000     .8904466    .9492836
      smb_a1 |   .8335145   .0166379    50.10   0.000     .8007277    .8663013
         hml |  -.0288262   .0177537    -1.62   0.106    -.0638118    .0061593
       _cons |   .0006457   .0006294     1.03   0.306    -.0005946    .0018861
------------------------------------------------------------------------------


. reg  s1_9_rf  rm_rf smb_a1 hml if year>2000 | year < 1991

      Source |       SS       df       MS              Number of obs =     219
-------------+------------------------------           F(  3,   215) = 2383.21
       Model |  .639168969     3  .213056323           Prob > F      =  0.0000
    Residual |  .019220754   215  .000089399           R-squared     =  0.9708
-------------+------------------------------           Adj R-squared =  0.9704
       Total |  .658389723   218  .003020136           Root MSE      =  .00946

------------------------------------------------------------------------------
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9706231   .0129148    75.16   0.000     .9451672     .996079
      smb_a1 |   .8259398   .0189739    43.53   0.000     .7885411    .8633386
         hml |  -.0523913   .0231754    -2.26   0.025    -.0980714   -.0067113
       _cons |   .0011646    .000666     1.75   0.082    -.0001482    .0024774
------------------------------------------------------------------------------


. sum smb_a1 if year>1989

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      smb_a1 |       228    .0019709     .037786  -.1436928   .1198193

. sum smb_a1 if year<2001

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      smb_a1 |       243    .0024844    .0343795   -.082738   .1198193

. sum smb_a1 if year>2000 | year < 1991

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
      smb_a1 |       219    .0020659    .0349835  -.1436928   .1099473
```

### A1.1.2 Check of sensitivity to the SMB definition

Another potential sensitivity is the sensitivity to the definition of the small-minus-big factor. To test this, we defined the SMB parameter on a range of differences in size and based on equally weighted and value weighted portfolios. The different SMB factor definitions are based on the differences between the value-weighted returns in the smallest/smaller less the larger size-based decile portfolios. We start with an alternative where the SMB is formed using only the odd-numbered deciles. We also consider the SMB according to Gregory et al 2009, based on the smallest (first) decile minus the largest (tenth) decile, and the smallest 5 deciles less the largest 5 deciles. In general, similar regressions statistics and similar parameter estimates for the SMB coefficient are found. We conclude that the definition of the SMB factor does not create a particular sensitivity to the finding of the small company premium.

```
.
. reg  s1_9_rf  rm_rf smb_a2 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 2256.26
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9632
                                                       Root MSE      =  .00988

------------------------------------------------------------------------------
             |               Robust
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9516627   .0128535    74.04   0.000      .926379    .9769463
      smb_a2 |   .8462112    .019434    43.54   0.000     .8079831    .8844394
         hml |   -.020157   .0227364    -0.89   0.376    -.0648811    .0245671
       _cons |   .0004943   .0005534     0.89   0.372    -.0005944    .0015829
------------------------------------------------------------------------------

.
. reg  s1_9_rf  rm_rf smb_g hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 1015.94
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9344
                                                       Root MSE      =  .01319

------------------------------------------------------------------------------
             |               Robust
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .8813218   .0212619    41.45   0.000     .8394981    .9231455
       smb_g |   .9359854   .0299391    31.26   0.000     .8770932    .9948777
         hml |   .1276762   .0234453     5.45   0.000     .0815576    .1737948
       _cons |   .0029088   .0007352     3.96   0.000     .0014625    .0043551
------------------------------------------------------------------------------



. reg  s1_9_rf  rm_rf smb_a1_10 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) =  624.36
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.8797
                                                       Root MSE      =  .01786

------------------------------------------------------------------------------
             |               Robust
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.029891   .0244736    42.08   0.000     .9817499    1.078033
   smb_a1_10 |   .5367902   .0253984    21.13   0.000     .4868298    .5867506
         hml |   .0130427   .0376429     0.35   0.729    -.0610034    .0870889
       _cons |  -.0033247   .0010209    -3.26   0.001    -.0053329   -.0013165
------------------------------------------------------------------------------

.
. reg  s1_9_rf  rm_rf smb_a1_8 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 1691.79
                                                       Prob > F      =  0.0000
```

```
                                                       R-squared     =  0.9366
                                                       Root MSE      =  .01297

------------------------------------------------------------------------------
             |               Robust
    s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.071797    .017179    62.39   0.000     1.038005     1.10559
    smb_a1_8 |    .993158   .0280291    35.43   0.000     .9380229    1.048293
         hml |   .0197644   .0345108     0.57   0.567    -.0481207    .0876494
       _cons |  -.0005046   .0006939    -0.73   0.468    -.0018695    .0008603
------------------------------------------------------------------------------

. reg  s2e_8_rf  rm_rf smb_a2 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,    335) = 1597.35
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9409
                                                       Root MSE      =  .01273

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9580437   .0151977    63.04   0.000     .9281487    .9879387
      smb_a2 |   .8468263   .0230782    36.69   0.000     .8014298    .8922229
         hml |  -.0147196   .0301596    -0.49   0.626    -.0740456    .0446064
       _cons |  -.0003343   .0007113    -0.47   0.639    -.0017335    .0010649
------------------------------------------------------------------------------

.
. reg  s2e_8_rf  rm_rf smb_g hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,    335) = 1081.92
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9311
                                                       Root MSE      =  .01374

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .8876923   .0216921    40.92   0.000     .8450225    .9303621
       smb_g |   .9652193   .0290412    33.24   0.000     .9080932    1.022345
         hml |   .1341542   .0248193     5.41   0.000     .0853328    .1829755
       _cons |   .0020888   .0007586     2.75   0.006     .0005965    .0035811
------------------------------------------------------------------------------

.
. reg  s2e_8_rf  rm_rf smb_a1_10 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,    335) =  454.58
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.8475
                                                       Root MSE      =  .02045

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.032341   .0283063    36.47   0.000      .976661    1.088022
```

```
    smb_a1_10 |   .5229003   .0307441    17.01   0.000     .4624245    .5833761
          hml |   .0207401   .0428002     0.48   0.628    -.063451     .1049311
        _cons |  -.0039961    .001177    -3.40   0.001    -.0063113   -.0016809
------------------------------------------------------------------------------

. reg  s1o_9_rf  rm_rf smb_a3 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 1089.16
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9363
                                                       Root MSE      =  .01293

------------------------------------------------------------------------------
             |               Robust
    s1o_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9370303   .0170675    54.90   0.000     .9034572    .9706033
      smb_a3 |   .7777934   .0277206    28.06   0.000      .723265    .8323217
         hml |  -.0172824   .0267458    -0.65   0.519    -.0698933    .0353285
       _cons |    .001909   .0007064     2.70   0.007     .0005195    .0032986
------------------------------------------------------------------------------

.
. reg  s1o_9_rf  rm_rf smb_g hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) =  815.22
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9216
                                                       Root MSE      =  .01434

------------------------------------------------------------------------------
             |               Robust
    s1o_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .8762254   .0220502    39.74   0.000      .832851    .9195997
       smb_g |   .9125983   .0338294    26.98   0.000     .8460536    .9791431
         hml |   .1224938    .025982     4.71   0.000     .0713853    .1736023
       _cons |   .0035649   .0008012     4.45   0.000     .0019888    .0051409
------------------------------------------------------------------------------

.
. reg  s1o_9_rf  rm_rf smb_a1_10 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) =  777.79
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.8918
                                                       Root MSE      =  .01685

------------------------------------------------------------------------------
             |               Robust
    s1o_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.027931   .0225078    45.67   0.000     .9836566    1.072206
   smb_a1_10 |   .5479021   .0226979    24.14   0.000     .5032537    .5925505
         hml |   .0068848   .0358979     0.19   0.848    -.0637289    .0774986
       _cons |  -.0027876   .0009526    -2.93   0.004    -.0046615   -.0009137
------------------------------------------------------------------------------
```

### A1.1.3 Check of sensitivity to OLS assumptions

Another factor that might be considered a sensitivity is the correctness of the OLS or other modelling assumptions. While the standard CAPM methodology essentially defines the CAPM beta under OLS assumptions, we still think it is pertinent to test these assumptions, because they will potentially impact the statistical significance parameters reported, and thus the interpretation of the model. We tested for various forms of autocorrelation, ran ARIMA models, and used robust standard errors. In general, the significance of the SMB parameter is not changing in terms of sign, significance, or order of magnitude. We conclude that the finding of the small company premium is not particularly sensitive to the OLS assumptions.

```
. reg  s1_9_rf  rm_rf smb_a1 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 2084.78
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9679
                                                       Root MSE      =  .00923

------------------------------------------------------------------------------
             |               Robust
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9497979   .0129324    73.44   0.000     .9243589    .9752368
      smb_a1 |   .8457304   .0197472    42.83   0.000     .8068863    .8845745
         hml |  -.0211472   .0198125    -1.07   0.287    -.0601198    .0178254
       _cons |   .0007738   .0005178     1.49   0.136    -.0002447    .0017923
------------------------------------------------------------------------------

.
. reg  s2e_8_rf  rm_rf smb_a2 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 1597.35
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9409
                                                       Root MSE      =  .01273

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9580437   .0151977    63.04   0.000     .9281487    .9879387
      smb_a2 |   .8468263   .0230782    36.69   0.000     .8014298    .8922229
         hml |  -.0147196   .0301596    -0.49   0.626    -.0740456    .0446064
       _cons |  -.0003343   .0007113    -0.47   0.639    -.0017335    .0010649
```

```
Prais-Winsten AR(1) regression -- iterated estimates

Regression with robust standard errors                 Number of obs =     339
                                                       F(  4,   335) = 1592.76
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9662
                                                       Root MSE      =  .00915

------------------------------------------------------------------------------
             |             Semi-robust
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
```

```
-------------+----------------------------------------------------------------
       rm_rf |   .9447138   .0129093    73.18   0.000     .9193203    .9701074
      smb_a1 |   .8405183   .0201346    41.74   0.000     .8009121    .8801245
         hml |  -.0195026   .0201958    -0.97   0.335    -.0592291    .0202239
       _cons |   .0008004   .0005921     1.35   0.177    -.0003642    .0019651
-------------+----------------------------------------------------------------
         rho |    .133213
------------------------------------------------------------------------------
Durbin-Watson statistic (original)    1.730626
Durbin-Watson statistic (transformed) 1.985349

.
. reg  d1.s1_9_rf  d1.rm_rf d1.smb_a1 d1.hml, robust

Regression with robust standard errors                 Number of obs =     338
                                                       F(  3,    334) = 1579.86
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9601
                                                       Root MSE      =  .01204

------------------------------------------------------------------------------
             |               Robust
D.s1_9_rf    |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
rm_rf        |
          D1 |   .9258575   .0137252    67.46   0.000     .8988588    .9528561
smb_a1       |
          D1 |   .8156856   .0207658    39.28   0.000     .7748373    .8565338
hml          |
          D1 |  -.0165498   .0226402    -0.73   0.465    -.0610851    .0279855
_cons        |   .0000933   .0006551     0.14   0.887    -.0011954    .0013819
------------------------------------------------------------------------------


. prais  s2e_8_rf  rm_rf smb_a2 hml, robust


Prais-Winsten AR(1) regression -- iterated estimates

Regression with robust standard errors                 Number of obs =     339
                                                       F(  4,    335) = 1228.80
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9404
                                                       Root MSE      =  .01272

------------------------------------------------------------------------------
             |             Semi-robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9575215   .0151834    63.06   0.000     .9276548    .9873883
      smb_a2 |    .846096   .0231404    36.56   0.000     .8005772    .8916147
         hml |  -.0146543   .0302856    -0.48   0.629    -.0742281    .0449196
       _cons |  -.0003312   .0007229    -0.46   0.647    -.0017532    .0010907
-------------+----------------------------------------------------------------
         rho |   .0166966
------------------------------------------------------------------------------
Durbin-Watson statistic (original)    1.961760
Durbin-Watson statistic (transformed) 1.993761

.
. reg  d1.s2e_8_rf  d1.rm_rf d1.smb_a2 d1.hml, robust

Regression with robust standard errors                 Number of obs =     338
```

```
                                                      F(  3,   334) = 1050.49
                                                      Prob > F      =  0.0000
                                                      R-squared     =  0.9174
                                                      Root MSE      =  .01776

------------------------------------------------------------------------------
             |               Robust
D.s2e_8_rf   |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
rm_rf        |
          D1 |   .9362462   .0172036    54.42   0.000     .9024053    .9700872
smb_a2       |
          D1 |   .8095473   .0259707    31.17   0.000     .7584605     .860634
hml          |
          D1 |   -.017054   .0333694    -0.51   0.610    -.0826946    .0485867
_cons        |    .000045   .0009669     0.05   0.963    -.0018569     .001947
------------------------------------------------------------------------------

.
. reg  s1_9_rf  rm_rf smb_a1 hml umd, robust

Regression with robust standard errors                Number of obs =     339
                                                      F(  4,   334) = 1399.77
                                                      Prob > F      =  0.0000
                                                      R-squared     =  0.9689
                                                      Root MSE      =   .0091

------------------------------------------------------------------------------
             |               Robust
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9448232   .0135642    69.66   0.000     .9181411    .9715052
      smb_a1 |   .8451908   .0197906    42.71   0.000     .8062608    .8841208
         hml |  -.0454588   .0204556    -2.22   0.027    -.0856968   -.0052208
         umd |  -.0460373   .0195115    -2.36   0.019    -.0844182   -.0076564
       _cons |   .0013151   .0005392     2.44   0.015     .0002545    .0023757
------------------------------------------------------------------------------

.
.
. arima s2e_8_rf  rm_rf smb_a2 hml umd, arima(0,1,1) robust

ARIMA regression

Sample:  2 to 339                              Number of obs      =       338
                                               Wald chi2(5)       =   5823.78
Log pseudo-likelihood =  999.8842              Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |             Semi-robust
D.s2e_8_rf   |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
s2e_8_rf     |
rm_rf        |
          D1 |   .9567765   .0151052    63.34   0.000     .9271708    .9863822
smb_a2       |
          D1 |   .8499656   .0216752    39.21   0.000      .807483    .8924482
hml          |
          D1 |  -.0386153   .0297294    -1.30   0.194    -.0968839    .0196533
umd          |
          D1 |  -.0478231   .0243515    -1.96   0.050    -.0955512    -.000095
_cons        |   .0000172   .0000124     1.39   0.165    -7.08e-06    .0000414
-------------+----------------------------------------------------------------
ARMA         |
```

```
ma           |
          L1 |  -.9871338   .0245196   -40.26   0.000    -1.035191   -.9390763
-------------+----------------------------------------------------------------
      /sigma |   .0124928   .0004878    25.61   0.000     .0115367    .0134489
------------------------------------------------------------------------------

. corrgram e2

                                          -1       0       1 -1       0       1
 LAG       AC       PAC      Q     Prob>Q  [Autocorrelation]  [Partial Autocor]
-------------------------------------------------------------------------------
1       -0.0261  -0.0262   .23318  0.6292          |                  |
2       -0.0383  -0.0397   .73377  0.6929          |                  |
3        0.0203   0.0187   .87471  0.8315          |                  |
4        0.0258   0.0252   1.1029  0.8938          |                  |
5       -0.0090  -0.0063   1.1307  0.9514          |                  |
6       -0.0316  -0.0317   1.4756  0.9611          |                  |
7        0.0427   0.0431   2.1099  0.9535          |                  |
8        0.0157   0.0168   2.1954  0.9744          |                  |
9       -0.0769  -0.0748   4.2611  0.8934          |                  |
10       0.0275   0.0258   4.5256  0.9205          |                  |
11       0.0416   0.0365   5.1348  0.9245          |                  |
12      -0.0423  -0.0385   5.7653  0.9274          |                  |
13       0.0114   0.0173    5.811  0.9528          |                  |
14       0.0374   0.0347   6.3076  0.9581          |                  |
15      -0.0207  -0.0254   6.4595  0.9710          |                  |
16       0.0391   0.0571   7.0048  0.9732          |                  |
17      -0.0324  -0.0394   7.3807  0.9781          |                  |
18      -0.1268  -0.1542   13.158  0.7821         -|                 -|
19       0.0009   0.0019   13.159  0.8303          |                  |
20      -0.0582  -0.0769   14.385  0.8105          |                  |
21      -0.0310  -0.0488   14.733  0.8361          |                  |
22       0.0458   0.0566   15.496  0.8401          |                  |
23       0.0164   0.0222   15.594  0.8721          |                  |
24       0.0435   0.0463   16.287  0.8774          |                  |
25       0.0366   0.0760    16.78  0.8896          |                  |
26      -0.0394  -0.0516   17.353  0.8980          |                  |
27      -0.0043  -0.0268    17.36  0.9219          |                  |
28      -0.0274  -0.0115   17.637  0.9349          |                  |
29       0.1321   0.1468   24.127  0.7227          |-                 |-
30       0.0012  -0.0166   24.127  0.7662          |                  |
31      -0.1030  -0.0919   28.101  0.6160          |                  |
32      -0.0804  -0.1095    30.53  0.5410          |                  |
33       0.0201   0.0098   30.682  0.5830          |                  |
34       0.0482   0.0975   31.561  0.5877          |                  |
35       0.0246   0.0189    31.79  0.6239          |                  |
36       0.1132   0.1034   36.665  0.4378          |                  |
37       0.0678   0.0885   38.419  0.4051          |                  |
38       0.0710   0.1108   40.348  0.3669          |                  |
39      -0.0669  -0.1014   42.067  0.3396          |                  |
40      -0.0323  -0.0612   42.469  0.3651          |                  |
```

## A1.1.4 Check of sensitivity to the dependent variable portfolio definition

Another potential form of sensitivity could be the definition of the dependent variable portfolio, so we check the sensitivity of this by using various forms of portfolios.

A key point to recall here is that it is necessary to form the dependent variable portfolios with some relationship to size. Ideally, we would like to consider a single, liquidly traded, but still small company, stock, as the particular stock in question for the inclusion of the small company premium.

However, using proxy data is very often fundamentally flawed for this exercise because a) most utility and liquidly traded UK and US stocks are big not small; and b) small stocks are often so illiquid, that neither CAPM nor the FFTM have very much explanatory power, and it is unlikely that the assumptions of CAPM or other factor models would hold. Thus the results of such regressions are really simply verifying that rejecting the foundational assumptions of the model, for some small stocks, implies that the model should not be used—but there is no difference between CAPM and the FFTM along these lines.

The test of the model is then to regress portfolios with some sortation on size on the FFTM factors. Below we use a variety of size-based portfolios as the dependent variables. The dependent variables are the average equally weighted returns from the 10 decile size-based portfolios from Gregory et al 2009. We then took the average return for the first (smallest) 3, the first 5, the first 8, and the first 9 deciles as the dependent variable.

```
. reg  s1_3_rf  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) =  938.41
       Model |  .807850849     3  .269283616           Prob > F      =  0.0000
    Residual |   .09613096   335  .000286958           R-squared     =  0.8937
-------------+------------------------------           Adj R-squared =  0.8927
       Total |  .903981809   338  .002674502           Root MSE      =  .01694

------------------------------------------------------------------------------
     s1_3_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .8197835   .0200257    40.94   0.000     .7803916    .8591754
      smb_a1 |   .9913078   .0262436    37.77   0.000     .9396847    1.042931
         hml |   .0031578   .0282662     0.11   0.911    -.0524437    .0587593
       _cons |   .0054732   .0009369     5.84   0.000     .0036303     .007316
------------------------------------------------------------------------------
```

```
. reg  s1_5_rf  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1608.17
       Model |  .832856645     3  .277618882           Prob > F      =  0.0000
    Residual |  .057831294   335  .000172631           R-squared     =  0.9351
-------------+------------------------------           Adj R-squared =  0.9345
       Total |   .89068794   338  .002635171           Root MSE      =  .01314

------------------------------------------------------------------------------
     s1_5_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .8622714   .0155323    55.51   0.000     .8317182    .8928246
      smb_a1 |   .9640008   .0203551    47.36   0.000     .9239608    1.004041
         hml |  -.0210299   .0219238    -0.96   0.338    -.0641556    .0220958
       _cons |   .0030157   .0007266     4.15   0.000     .0015864    .0044451
------------------------------------------------------------------------------
```

```
. reg  s1_8_rf  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 2965.39
       Model |  .856877455     3  .285625818           Prob > F      =  0.0000
```

```
    Residual |  .032267187   335   .00009632           R-squared     =  0.9637
-------------+------------------------------           Adj R-squared =  0.9634
       Total |  .889144642   338  .002630605           Root MSE      =  .00981

------------------------------------------------------------------------------
     s1_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9233073   .0116021    79.58   0.000     .9004852    .9461295
      smb_a1 |   .8941556   .0152045    58.81   0.000     .8642472    .9240639
         hml |  -.0252549   .0163763    -1.54   0.124    -.0574682    .0069584
       _cons |   .0011538   .0005428     2.13   0.034     .0000861    .0022215
------------------------------------------------------------------------------
```

```
. reg  s1_9_rf  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 3367.53
       Model |  .859778215     3  .286592738           Prob > F      =  0.0000
    Residual |  .028510046   335  .000085105           R-squared     =  0.9679
-------------+------------------------------           Adj R-squared =  0.9676
       Total |  .888288261   338  .002628072           Root MSE      =  .00923

------------------------------------------------------------------------------
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9497979   .0109057    87.09   0.000     .9283455    .9712502
      smb_a1 |   .8457304   .0142919    59.18   0.000     .8176171    .8738436
         hml |  -.0211472   .0153934    -1.37   0.170    -.0514271    .0091327
       _cons |   .0007738   .0005102     1.52   0.130    -.0002298    .0017774
------------------------------------------------------------------------------
```

We note that there has been in the literature a possible criticism of the FFTM that there is a potential for some spuriousness in the regression of the portfolio formed on size on the SMB factor. As a test of this, what we do is to define the SMB factor based on the odd-number deciles of the 10 value-weighted deciles formed on size; while we form the dependent variable portfolio returns based on the even-deciles from the equally-weighted decile portfolios formed on size. Thus, in general, no decile is found in the dependent variable that was used in the SMB definition.

```
. reg  s2e_8_rf  rm_rf smb_a2 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1779.23
       Model |  .864327124     3  .288109041           Prob > F      =  0.0000
    Residual |  .054246281   335  .000161929           R-squared     =  0.9409
-------------+------------------------------           Adj R-squared =  0.9404
       Total |  .918573405   338  .002717673           Root MSE      =  .01273

------------------------------------------------------------------------------
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9580437   .0150493    63.66   0.000     .9284407    .9876468
      smb_a2 |   .8468263   .0198655    42.63   0.000     .8077495    .8859032
         hml |  -.0147196    .021233    -0.69   0.489    -.0564863    .0270471
       _cons |  -.0003343   .0007042    -0.47   0.635    -.0017196     .001051
```

```
. reg  s2e_8_rf  rm_rf smb_a2 hml, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 1597.35
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9409
                                                       Root MSE      =  .01273

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9580437   .0151977    63.04   0.000     .9281487    .9879387
      smb_a2 |   .8468263   .0230782    36.69   0.000     .8014298    .8922229
         hml |  -.0147196   .0301596    -0.49   0.626    -.0740456    .0446064
       _cons |  -.0003343   .0007113    -0.47   0.639    -.0017335    .0010649
------------------------------------------------------------------------------
```

## A1.1.5 Check of sensitivity to adding or subtracting factors

Another check of sensitivity is to check whether adding or subtracting factors impacts the SMB coefficients. One way around this is to estimate "simple" (i.e., single factor) regression coefficients. We also tested by adding additional factors and dropping the HML factor. The results of the regressions are found below. We conclude that the finding of a small company premium is not sensitive to the adding or subtracting of factors.

```
. *****testing adding and subtracting factors for two different portfolios and
two different SMBs
.
. reg  s1_9_rf  rm_rf smb_a1 hml umd, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  4,   334) = 1399.77
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9689
                                                       Root MSE      =   .0091

------------------------------------------------------------------------------
             |               Robust
     s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9448232   .0135642    69.66   0.000     .9181411    .9715052
      smb_a1 |   .8451908   .0197906    42.71   0.000     .8062608    .8841208
         hml |  -.0454588   .0204556    -2.22   0.027    -.0856968   -.0052208
         umd |  -.0460373   .0195115    -2.36   0.019    -.0844182   -.0076564
       _cons |   .0013151   .0005392     2.44   0.015     .0002545    .0023757
------------------------------------------------------------------------------

.
. reg  s1_9_rf  rm_rf smb_a1 umd, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 1777.04
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9682
                                                       Root MSE      =  .00919
```

```
------------------------------------------------------------------------------
             |               Robust
    s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9472075   .0138335    68.47   0.000     .9199959     .974419
      smb_a1 |   .8410208   .0203782    41.27   0.000     .8009354    .8811062
         umd |  -.0286761   .0182141    -1.57   0.116    -.0645044    .0071523
       _cons |   .0009386   .0005164     1.82   0.070    -.0000772    .0019545
------------------------------------------------------------------------------

.
. reg  s1_9_rf  rm_rf smb_a1, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  2,   336) = 2942.75
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9677
                                                       Root MSE      =  .00924

------------------------------------------------------------------------------
             |               Robust
    s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9500941   .0133035    71.42   0.000     .9239254    .9762627
      smb_a1 |   .8431819   .0198516    42.47   0.000     .8041329    .8822309
       _cons |   .0006734   .0005175     1.30   0.194    -.0003446    .0016914
------------------------------------------------------------------------------

.
. reg  s1_9_rf  rm_rf smb_a1 hml umd, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  4,   334) = 1399.77
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9689
                                                       Root MSE      =   .0091

------------------------------------------------------------------------------
             |               Robust
    s1_9_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9448232   .0135642    69.66   0.000     .9181411    .9715052
      smb_a1 |   .8451908   .0197906    42.71   0.000     .8062608    .8841208
         hml |  -.0454588   .0204556    -2.22   0.027    -.0856968   -.0052208
         umd |  -.0460373   .0195115    -2.36   0.019    -.0844182   -.0076564
       _cons |   .0013151   .0005392     2.44   0.015     .0002545    .0023757
------------------------------------------------------------------------------

.
. reg  s2e_8_rf  rm_rf smb_a2 hml umd, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  4,   334) = 1148.29
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9419
                                                       Root MSE      =  .01264

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9530479   .0155357    61.35   0.000     .9224879     .983608
      smb_a2 |   .8462793   .0222473    38.04   0.000     .8025169    .8900417
```

```
         hml |  -.0391263   .0300067    -1.30   0.193    -.0981522    .0198996
         umd |  -.0462174   .0250133    -1.85   0.066    -.0954209    .0029862
       _cons |   .0002093   .0007424     0.28   0.778     -.001251    .0016696
------------------------------------------------------------------------------

.
. reg  s2e_8_rf  rm_rf smb_a2 umd, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  3,   335) = 1422.58
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9414
                                                       Root MSE      =  .01268

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9550905   .0161618    59.10   0.000     .9232992    .9868819
      smb_a2 |    .842668   .0222551    37.86   0.000     .7988907    .8864453
         umd |  -.0312741   .0253493    -1.23   0.218     -.081138    .0185898
       _cons |  -.0001135   .0007248    -0.16   0.876    -.0015391    .0013121
------------------------------------------------------------------------------

.
. reg  s2e_8_rf  rm_rf smb_a2, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  2,   336) = 2222.73
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9409
                                                       Root MSE      =  .01272

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9582451   .0154853    61.88   0.000     .9277847    .9887055
      smb_a2 |   .8450411   .0222279    38.02   0.000     .8013178    .8887645
       _cons |  -.0004036   .0007058    -0.57   0.568     -.001792    .0009848
------------------------------------------------------------------------------

.
. reg  s2e_8_rf  rm_rf smb_a2 hml umd, robust

Regression with robust standard errors                 Number of obs =     339
                                                       F(  4,   334) = 1148.29
                                                       Prob > F      =  0.0000
                                                       R-squared     =  0.9419
                                                       Root MSE      =  .01264

------------------------------------------------------------------------------
             |               Robust
    s2e_8_rf |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9530479   .0155357    61.35   0.000     .9224879     .983608
      smb_a2 |   .8462793   .0222473    38.04   0.000     .8025169    .8900417
         hml |  -.0391263   .0300067    -1.30   0.193    -.0981522    .0198996
         umd |  -.0462174   .0250133    -1.85   0.066    -.0954209    .0029862
       _cons |   .0002093   .0007424     0.28   0.778     -.001251    .0016696
```

## A1.1.6 Check of the portfolios sorted on size and value

An additional check of the models is to use the benchmark portfolios sorted on *both size and value,* (book to market ratio). This is because the size effect might be plausibly coming from some other factor, which is just correlated with value, for example. The definitions of the dependent variable portfolios come directly from the Gregory et al 2009 dataset of six basic portfolios sorted on size and book to market, small-low (sl), small-medium (sm), small-high (sh), big-low (bl), big-medium (bm), and big-high (bh). We also check this for both the value-weighted and the equally-weighted portfolios. The results are found below. We conclude that the small company premium is not sensitive to the cross-sorting with the value parameter. More study of this could be done, however, as Gregory et al have provided a number of more detailed portfolios sorted/broken on different characteristics.

```
. reg  sl  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1020.05
       Model |  1.13709212     3  .379030706           Prob > F      =  0.0000
    Residual |  .124479193   335   .00037158           R-squared     =  0.9013
-------------+------------------------------           Adj R-squared =  0.9004
       Total |  1.26157131   338   .00373246           Root MSE      =  .01928

------------------------------------------------------------------------------
          sl |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.109254   .0227879    48.68   0.000     1.064429     1.15408
      smb_a1 |   .7399184   .0298635    24.78   0.000     .6811748     .798662
         hml |   -.609919    .032165   -18.96   0.000    -.6731898   -.5466482
       _cons |   .0033846   .0010661     3.17   0.002     .0012875    .0054816
------------------------------------------------------------------------------

. reg  sm  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1101.14
       Model |   .85790188     3  .285967293           Prob > F      =  0.0000
    Residual |  .086999934   335  .000259701           R-squared     =  0.9079
-------------+------------------------------           Adj R-squared =  0.9071
       Total |  .944901814   338  .002795567           Root MSE      =  .01612

------------------------------------------------------------------------------
          sm |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.016037   .0190509    53.33   0.000      .9785627    1.053512
      smb_a1 |   .6717761   .0249662    26.91   0.000      .6226659    .7208863
         hml |   .0224799   .0268902     0.84   0.404     -.0304151    .0753749
       _cons |   .0037858   .0008912     4.25   0.000      .0020326    .0055389
------------------------------------------------------------------------------

. reg  sh  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1402.71
       Model |  .937217581     3   .31240586           Prob > F      =  0.0000
    Residual |  .074609699   335  .000222716           R-squared     =  0.9263
-------------+------------------------------           Adj R-squared =  0.9256
       Total |  1.01182728   338  .002993572           Root MSE      =  .01492

------------------------------------------------------------------------------
```

```
          sh |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.020807   .0176422    57.86   0.000     .9861033     1.05551
      smb_a1 |   .7036746   .0231201    30.44   0.000     .6581957    .7491535
         hml |   .3526438   .0249019    14.16   0.000       .30366    .4016276
       _cons |   .0052305   .0008253     6.34   0.000     .0036069     .006854
------------------------------------------------------------------------------

.
. reg  sl_ew  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1016.12
       Model |  1.10038566     3  .36679522            Prob > F      =  0.0000
    Residual |  .120926746   335  .000360975           R-squared     =  0.9010
-------------+------------------------------           Adj R-squared =  0.9001
       Total |  1.22131241   338  .00361335            Root MSE      =    .019

------------------------------------------------------------------------------
       sl_ew |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.038646   .0224604    46.24   0.000     .9944645    1.082827
      smb_a1 |   .9169853   .0294343    31.15   0.000      .859086    .9748846
         hml |  -.5454543   .0317027   -17.21   0.000    -.6078158   -.4830929
       _cons |   .0052515   .0010508     5.00   0.000     .0031846    .0073184
------------------------------------------------------------------------------

. reg  sm_ew  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1622.69
       Model |  .837115993     3  .279038664           Prob > F      =  0.0000
    Residual |  .057606655   335   .00017196           R-squared     =  0.9356
-------------+------------------------------           Adj R-squared =  0.9350
       Total |  .894722648   338  .002647108           Root MSE      =  .01311

------------------------------------------------------------------------------
       sm_ew |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9383188   .0155022    60.53   0.000      .907825    .9688127
      smb_a1 |   .8335949   .0203155    41.03   0.000     .7936328    .8735571
         hml |  -.0517244   .0218812    -2.36   0.019    -.0947663   -.0086825
       _cons |   .0060071   .0007252     8.28   0.000     .0045805    .0074337
------------------------------------------------------------------------------

. reg  sh_ew  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1755.58
       Model |  .817555842     3  .272518614           Prob > F      =  0.0000
    Residual |  .052002015   335   .00015523           R-squared     =  0.9402
-------------+------------------------------           Adj R-squared =  0.9397
       Total |  .869557857   338  .002572656           Root MSE      =  .01246

------------------------------------------------------------------------------
       sh_ew |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .8833222   .0147287    59.97   0.000     .8543497    .9122947
      smb_a1 |   .8665865    .019302    44.90   0.000     .8286181    .9045549
         hml |   .1923692   .0207896     9.25   0.000     .1514747    .2332637
       _cons |   .0090076    .000689    13.07   0.000     .0076522     .010363
------------------------------------------------------------------------------

.
```

```
. reg  bl  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1001.25
       Model |  .70712613     3  .23570871             Prob > F      =  0.0000
    Residual |  .07886355   335  .000235414            R-squared     =  0.8997
-------------+------------------------------           Adj R-squared =  0.8988
       Total |  .78598968   338  .002325413            Root MSE      =  .01534

------------------------------------------------------------------------------
          bl |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9036217   .0181382    49.82   0.000     .8679426    .9393007
      smb_a1 |  -.1524841   .0237701    -6.41   0.000    -.1992415   -.1057267
         hml |  -.4295718    .025602   -16.78   0.000    -.4799327   -.3792109
       _cons |   .0077381   .0008486     9.12   0.000      .006069    .0094073
------------------------------------------------------------------------------

. reg  bm  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) =  986.81
       Model |  .812228283     3  .270742761           Prob > F      =  0.0000
    Residual |  .091911152   335  .000274362           R-squared     =  0.8983
-------------+------------------------------           Adj R-squared =  0.8974
       Total |  .904139435   338  .002674969           Root MSE      =  .01656

------------------------------------------------------------------------------
          bm |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.052849   .0195812    53.77   0.000     1.014331    1.091367
      smb_a1 |  -.0610617   .0256612    -2.38   0.018     -.111539   -.0105844
         hml |   .0267905   .0276388     0.97   0.333     -.027577    .0811579
       _cons |   .0050127   .0009161     5.47   0.000     .0032107    .0068146
------------------------------------------------------------------------------

. reg  bh  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) =  854.67
       Model |  .840006483     3  .280002161           Prob > F      =  0.0000
    Residual |  .109750224   335  .000327613           R-squared     =  0.8844
-------------+------------------------------           Adj R-squared =  0.8834
       Total |  .949756707   338  .002809931           Root MSE      =   .0181

------------------------------------------------------------------------------
          bh |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   .9920694   .0213973    46.36   0.000     .9499794    1.034159
      smb_a1 |  -.1162403   .0280411    -4.15   0.000    -.1713991   -.0610814
         hml |   .6078655   .0302021    20.13   0.000     .5484558    .6672753
       _cons |   .0058922    .001001     5.89   0.000     .0039232    .0078613
------------------------------------------------------------------------------

.
. reg  bl_ew  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1025.88
       Model |  .971830834     3  .323943611           Prob > F      =  0.0000
    Residual |  .105783106   335   .00031577           R-squared     =  0.9018
-------------+------------------------------           Adj R-squared =  0.9010
       Total |  1.07761394   338  .003188207           Root MSE      =  .01777
```

```
------------------------------------------------------------------------------
       bl_ew |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.098136    .021007    52.27   0.000     1.056814    1.139459
      smb_a1 |   .1940948   .0275296     7.05   0.000      .139942    .2482475
         hml |  -.5190855   .0296513   -17.51   0.000    -.5774117   -.4607594
       _cons |   .0054851   .0009828     5.58   0.000      .003552    .0074183
------------------------------------------------------------------------------

. reg  bm_ew  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) = 1034.21
       Model |  .842634635     3  .280878212           Prob > F      =  0.0000
    Residual |  .090981532   335  .000271587           R-squared     =  0.9025
-------------+------------------------------           Adj R-squared =  0.9017
       Total |  .933616167   338  .002762178           Root MSE      =  .01648

------------------------------------------------------------------------------
       bm_ew |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |    1.08259   .0194819    55.57   0.000     1.044268    1.120913
      smb_a1 |   .1614646   .0255311     6.32   0.000     .1112432     .211686
         hml |   .1332988   .0274987     4.85   0.000      .079207    .1873906
       _cons |   .0048187   .0009114     5.29   0.000     .0030259    .0066115
------------------------------------------------------------------------------

. reg  bh_ew  rm_rf smb_a1 hml

      Source |       SS       df       MS              Number of obs =     339
-------------+------------------------------           F(  3,   335) =  747.68
       Model |  .979064485     3  .326354828           Prob > F      =  0.0000
    Residual |  .146224718   335  .000436492           R-squared     =  0.8701
-------------+------------------------------           Adj R-squared =  0.8689
       Total |   1.1252892   338  .003329258           Root MSE      =  .02089

------------------------------------------------------------------------------
       bh_ew |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       rm_rf |   1.114459   .0246982    45.12   0.000     1.065876    1.163042
      smb_a1 |   .2621555    .032367     8.10   0.000     .1984873    .3258237
         hml |   .5003371   .0348615    14.35   0.000     .4317622    .5689121
       _cons |    .005281   .0011554     4.57   0.000     .0030082    .0075539
------------------------------------------------------------------------------
```

# Annex 2 Technical Annex

This technical annex presents additional technical information relating to the empirical analysis of small company cost of debt.

## A2.1 Random effects and fixed effects regression results

Figure 29 and Figure 30 show the full regression results of the random effects and fixed effects estimations on the equation:

$$Spread_{ij} = \alpha + \beta_1.log(Term\ to\ maturity_{ij}) + \beta_2.log(Total\ company\ value_{ij})$$

*Where: Spread*$_{ij}$ is the spread between the rate on bond 'i' issued by company 'j' and the rate on an equivalent government bond; *Term to maturity* is the number of days to maturity for bond 'i' issued by company 'j'; *Total company value* is the sum of the total debt (bonds and loans) and market capitalisation of company 'j'.

**Figure 29:Random effects regression results**

```
Random-effects GLS regression                Number of obs      =      1071
Group variable (i): compid                   Number of groups   =       124

R-sq:  within  = 0.1055                      Obs per group: min =         1
       between = 0.1150                                     avg =       8.6
       overall = 0.1588                                     max =       478

Random effects u_i ~ Gaussian                Wald chi2(2)       =    126.43
corr(u_i, X)       = 0 (assumed)             Prob > chi2        =    0.0000

--------------------------------------------------------------------------------
         spread |      Coef.   Std. Err.       z     P>|z|      [95% Conf. Interval]
----------------+---------------------------------------------------------------
Log term to mat |   36.77262   3.490827    10.53   0.000     29.93073    43.61452
Log total value |   -47.8804   13.04008    -3.67   0.000    -73.43849   -22.32231
       constant |   1087.455   302.8061     3.59   0.000     493.9657    1680.944
----------------+---------------------------------------------------------------
        sigma_u |  196.87913
        sigma_e |   106.6323
            rho |  .77318911   (fraction of variance due to u_i)
--------------------------------------------------------------------------------
```

Note: na
***Source: LE Analysis***

**Figure 30:Fixed effects regression results**

```
Fixed-effects (within) regression               Number of obs      =      1071
Group variable (i): compid                      Number of groups   =       124

R-sq:  within  = 0.1064                         Obs per group: min =         1
       between = 0.1166                                        avg =       8.6
       overall = 0.1201                                        max =       478

                                                F(2,945)           =     56.24
corr(u_i, Xb)  = -0.9798                        Prob > F           =    0.0000

------------------------------------------------------------------------------
         spread |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
----------------+-------------------------------------------------------------
Log term to mat |   37.38186    3.53609    10.57   0.000     30.44236    44.32135
Log total value |  -573.0097   548.7438    -1.04   0.297    -1649.907    503.8877
       constant |   13964.91   13453.82     1.04   0.300    -12437.91    40367.73
----------------+-------------------------------------------------------------
        sigma_u |  795.71998
        sigma_e |   106.6323
            rho |  .98235884   (fraction of variance due to u_i)
------------------------------------------------------------------------------
F test that all u_i=0:     F(123, 945) =    12.34            Prob > F = 0.0000
```

Note:

***Source: LE Analysis***

### A2.1.1 Hausman test results

Figure 31 presents the results of the Hausman test of whether the random effects or the fixed effects model should be used:

**Figure 31:Hausman test results**

```
. hausman fe re

                        ---- Coefficients ----
                |      (b)          (B)            (b-B)     sqrt(diag(V_b-V_B))
                |       fe           re         Difference          S.E.
----------------+----------------------------------------------------------------
Log term to mat |    37.38186     36.77262        .6092319        .5639691
Log total value |   -573.0097     -47.8804       -525.1293        548.5889
---------------------------------------------------------------------------------
                           b = consistent under Ho and Ha; obtained from xtreg
            B = inconsistent under Ha, efficient under Ho; obtained from xtreg

    Test:  Ho:  difference in coefficients not systematic

                  chi2(2) = (b-B)'[(V_b-V_B)^(-1)](b-B)
                          =        1.86
                Prob>chi2 =      0.3946
```

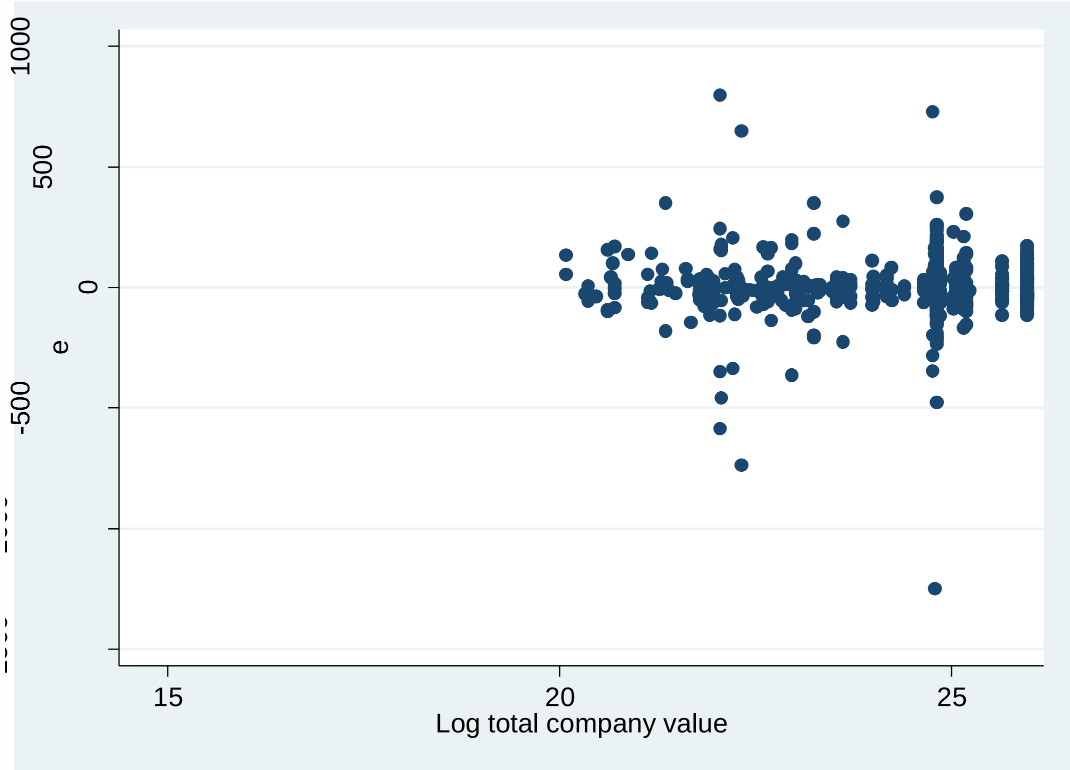
Note:
***Source:***

## A2.2 Plots of residuals

The figures below present the error components and residuals from the random effects model used in the analysis of small company cost of debt (Section 4.2), plotted against log of total company value and as histograms. Since we are using a random effects model we have two error components and the combined residual:

- $u_i$ = the random-error component
- $e_{ij}$ = the overall error component
- $ue_{ij}$ = the combined residual (=$u_i$ + $e_{ij}$)

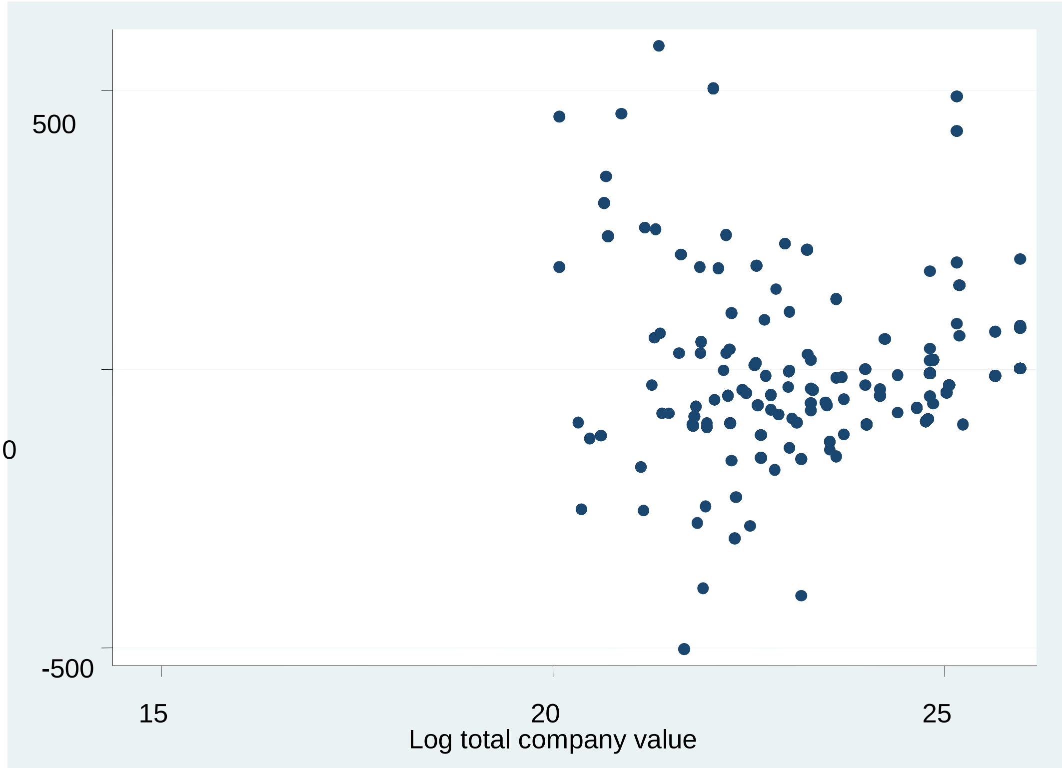
## Figure 32:Random-error component vs. log of total company value




Note:

***Source: LE Analysis***

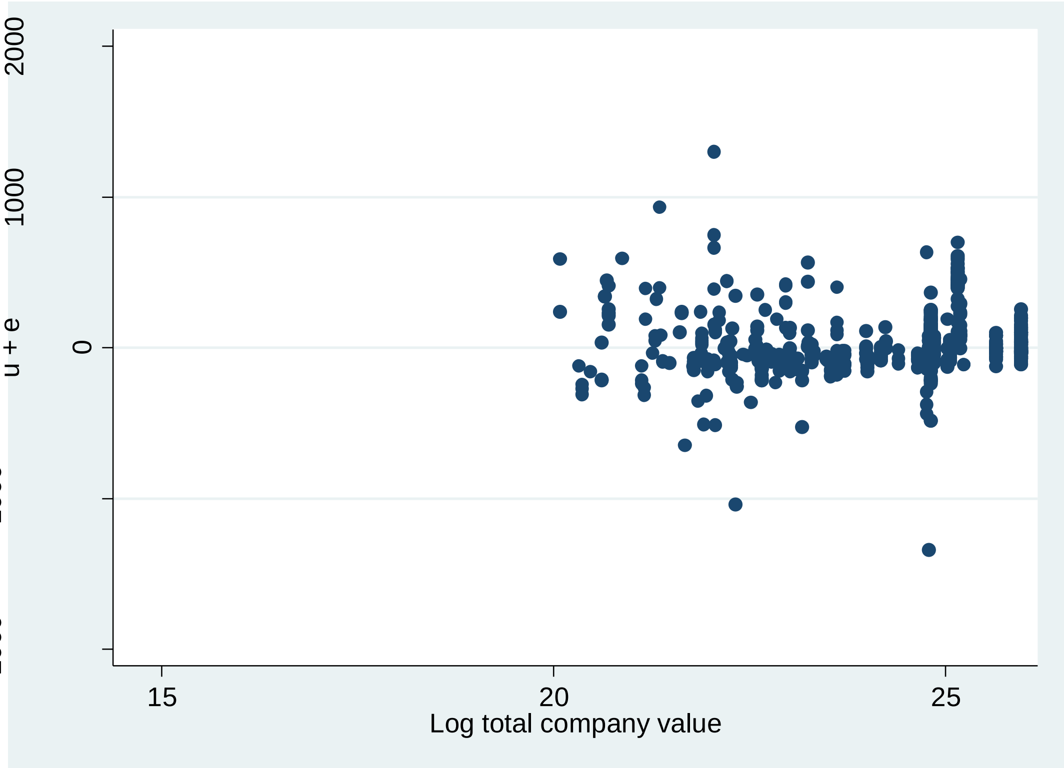
**Figure 33:Overall error component vs. log of total company value**




Note:

***Source: LE Analysis***

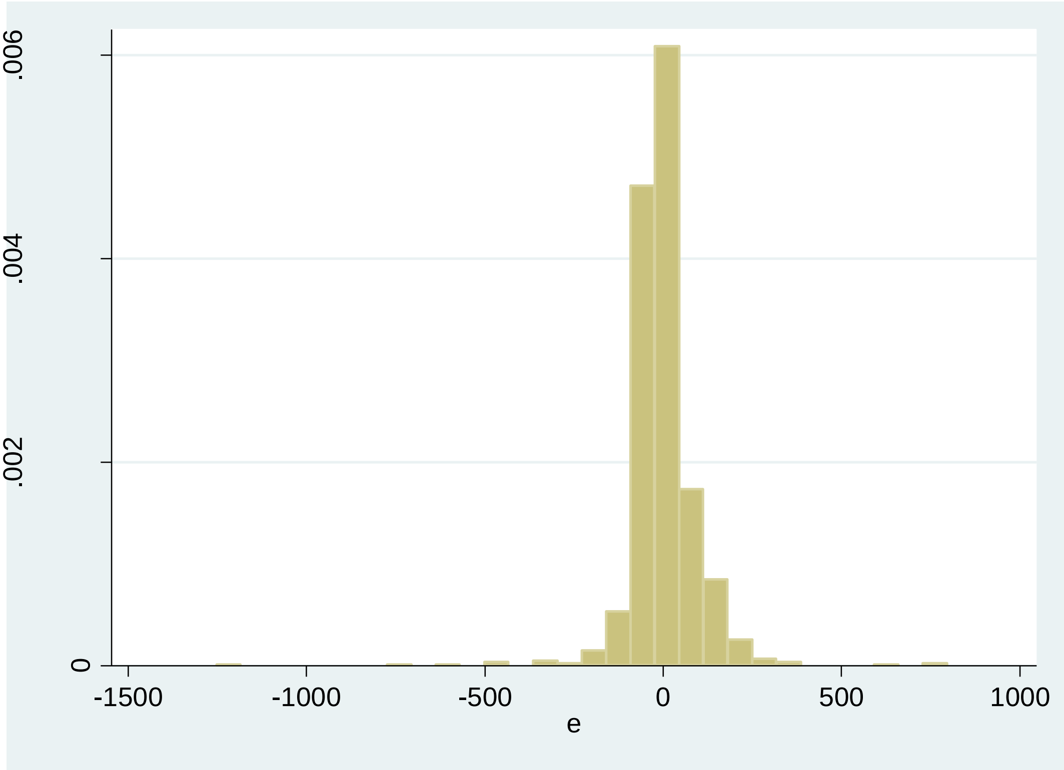
**Figure 34:Combined residual (=$u_i$ + $e_{ij}$) vs. log of total company value**




Note:

***Source: LE Analysis***

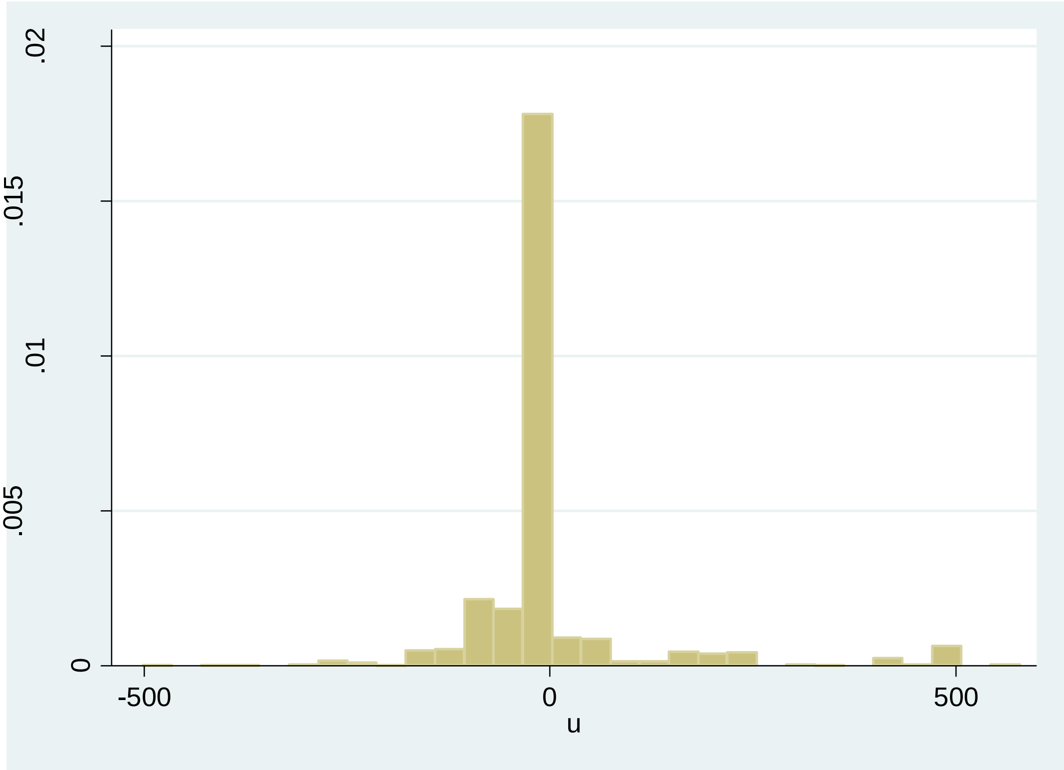
**Figure 35:Histogram of random-error component**




Note:

***Source: LE Analysis***

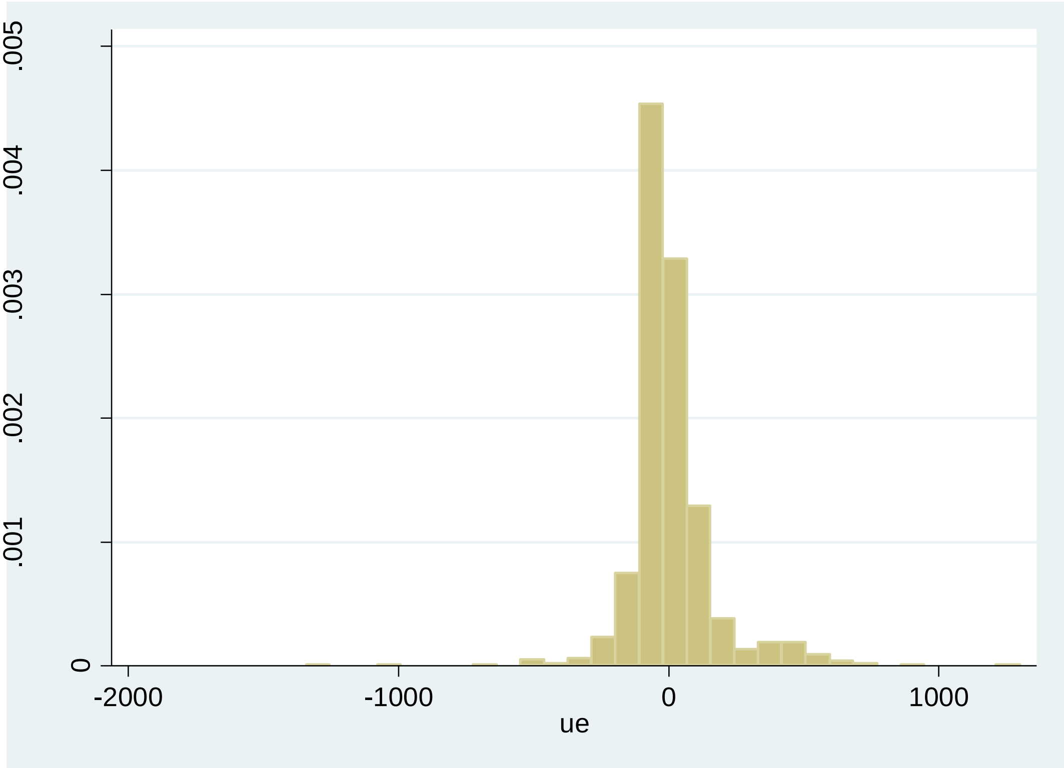
**Figure 36:Histogram of overall error component**




Note:

***Source: LE Analysis***

**Figure 37:Histogram of combined residual (=$u_i$ + $e_{ij}$)**




Note:
***Source: LE Analysis***

## A2.3 Additional statistical results

**Table 10: Test of the mean difference between smallest and largest quintile UK portfolio returns**

| t-Test: Two-Sample Assuming Unequal Variances | | |
|---|---|---|
| | | |
| | *Sv1* | *Sv5* |
| Mean | 0.018 | 0.010 |
| Variance | 0.003 | 0.002 |
| Observations | 339 | 339 |
| Hypothesized Mean Difference | - | |
| df | 663 | |

| | | |
|---|---|---|
| t Stat | 2.120 | |
| P(T<=t) one-tail | 0.017 | |
| t Critical one-tail | 1.647 | |
| P(T<=t) two-tail | 0.034 | |
| t Critical two-tail | 1.964 | |

### A2.3.1 Test of CAPM versus 3-factor model using UK data, residual sums of squares

F-Test Two-variances

| | *Residuals* | *Residuals* |
|---|---|---|
| Mean | -1.2E-17 | -1.7E-18 |
| Variance | 0.002052 | 0.000253 |
| Observations | 339 | 339 |
| df | 338 | 338 |
| F | 8.099297 | |
| P(F<=f) one-tail | 3.75E-71 | |
| F Critical one-tail | 1.196246 | |