The Concept of the Reference Node Invariance Threshold: and why it implies the existing NG transmission model is likely to lead to sub-optimal location of generation capacity.

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October 2010

Introduction1: Background – the National Grid Transmission Model.

1.1) The NG transmission model calculates the marginal cost of an extra unit of generation capacity at any node in the transmission network. The model starts by calculating what the transmission cost is, in terms of the required network capacity, for a base position representing the peak generation capacity or demand at each node in the system. The model then calculates what the change in transmission capacity would be if an extra MW of generation were put in at any particular node: the change is the marginal transmission cost of that node. Since inputs to the system must equal outputs, however, the extra MW of generation input at the node has to be balanced by an extra MW of demand at some chosen reference node.
(A detailed description of the model can be found in the user guide to "National Grid's Stand-Alone DCLF ICRP TNUoS Great Britain Transport & Tariff Model: Model Methodology & Operation", Version 3.2, April 2006. I am grateful to NG for making the documentation and the model available to me.)

1.2) It is stated in the User Guide that choice of reference node does not matter, in the sense that changing the reference node does not alter the relativity between the calculated marginal costs. Witness the following quotation from page 2 of the User Guide:-
"A reference node is required as a basis point for the calculation of marginal costs. It determines the magnitude of the marginal costs but not the relativity. For example, if the reference point were put in the North all nodal generation marginal costs would likely be negative. Conversely, if the reference point were defined at Land's End, all nodal generation marginal costs would be positive. However the relativity of costs between nodes would stay the same." (my underline)
I will call this property "reference node invariance", and denote it by RNI.

1.3) The marginal costs calculated by the transmission model form an important input to the process of calculating the connection costs which generators pay for use of the network. Since demand is equivalent to negative generation, they also play a part in determining the charges for users. The basic requirement is that charges should give appropriate signals so that optimal decisions are likely to be made about the location of generation capacity. As stated in NG's 1992 document "Proposed Investment Cost Pricing for Use of System", the charges *"provide appropriate signals for efficient economic decisions by the user, whether these are in terms of increasing or reducing the levels of demand or generation."*

1.4) This paper will be concerned with examining in more detail the circumstances under which RNI will hold: and the likely implications if the model is being applied in circumstances when RNI cannot be assumed. In particular, a key purpose of this paper is to examine to what extent the marginal costs calculated by the NG transmission model are likely to fulfil the requirement of giving accurate price signals for the optimal location of generation capacity.

Introduction 2: What this paper does.

2.1) As already noted, this paper is concerned with examining the circumstances under which reference node invariance, (RNI), will hold for a given network. The most important step in doing this is the introduction of the concept of the RNI threshold for a given network. Each network will have its own RNI threshold. The RNI threshold places a limit on the extent to which marginal costs, (as calculated by the NG transmission model), can be scaled up while still giving accurate signals about the resulting change in the system's required transmission capacity.

2.2) The paper will demonstrate how, for scaling factors below the RNI threshold, unit marginal costs as calculated by the NG transmission model can be scaled up, and will still give an accurate estimate of the resulting change to the transmission requirement. But for scaling factors above the RNI threshold, this is no longer the case: the required change to transmission capacity will no longer be independent of the original choice of reference node: and the scaled marginal cost will no longer give an accurate signal about the required change to transmission capacity.

2.3) The structure of the paper is as follows:-
Section 3 introduces necessary notation, and defines the concept of the RNI threshold for a network. An annex gives the proof of the basic result that RNI always holds for generation or demand increments below the RNI threshold, and that marginal costs can also be scaled within this range.
Section 4 gives a worked example for a simple transmission network. This illustrates several important features which occur as generation increments move above the RNI threshold.
Section 5 discusses resulting implications for the operation of the NG pricing model. In particular, since increments of generation capacity will commonly be above the RNI threshold for the system, this means that scaled up marginal costs from the NG transmission model will not give optimal price signals about the location of generation capacity.

Section 3: Definition of Reference Node Invariance Threshold.

3.1) Consider a transmission network with n nodes: and let the flow from node a to node b in the base or equilibrium state of the system be e_{ab}.
Suppose that the equilibrium position of the system is disturbed by putting an extra ε increment of generation in at node a, and also a compensating ε increment in demand at node x.
Let $m_{ax}(\varepsilon)$ denote the resulting change in transmission cost (positive or negative) from the base transmission cost of the system.
Then we define the network to be "reference node invariant at scale ε" if changing the reference node x does not alter the relativity between the marginal costs m. More formally

network is reference node invariant at scale ε if, and only if,

$$m_{ax}(\varepsilon) - m_{bx}(\varepsilon) = m_{ay}(\varepsilon) - m_{by}(\varepsilon)$$, for all nodes a, b, x and y.

Note that RNI as defined for the NG transmission model corresponds, in the notation used here, to reference node invariance at scale 1.

3.2) An easily proved consequence of the above definition, (which will be required later), is that an equivalent condition for reference node invariance at scale ε to hold is that

$$m_{ab}(\varepsilon) = m_{ac}(\varepsilon) + m_{cb}(\varepsilon) \quad \text{for all nodes a, b and c.}$$

3.3) Now suppose that the equilibrium network is disturbed by putting an extra ε units of generation in at node x, compensated by an extra ε units of demand at node y: and let $\delta_{ab}^{(xy)}(\varepsilon)$ denote the resulting change in transmission flow on link ab. Then if e_{ab} and (e_{ab} + $\delta_{ab}^{(xy)}(\varepsilon)$) have the same signs, then the disturbance to the network has not altered the direction of flow on link ab: whereas if e_{ab} and (e_{ab} + $\delta_{ab}^{(xy)}(\varepsilon)$) have different signs, then the direction of flow on link ab has been changed.

If e_{ab} and (e_{ab} + $\delta_{ab}^{(xy)}(\varepsilon)$) have the same signs for all possible a, b, x and y, then putting in an extra ε of generation at any single node x, and an extra ε of demand at any single node y, does not change any of the flow directions in the equilibrium system.

Define the quantity Δ to be the largest ε with this property: more formally

$$\Delta = \sup\{\varepsilon : e_{ab} \text{ and } (e_{ab} + \delta_{ab}^{(xy)}(\varepsilon)) \text{ have the same signs for all a, b, x and y}\}.$$

3.4) The following result establishes the link between this quantity Δ and the concept of reference node invariance at scale ε.

Theorem

For Δ as defined above

(i) the network is reference node invariant at scale ε for all $0 \le \varepsilon \le \Delta$.

(ii) $m_{ax}(\varepsilon) = \varepsilon\, m_{ax}(1)$ for all $0 \le \varepsilon \le \Delta$, provided $\Delta \ge 1$.

Proof

The proof is given in the annex.

3.5) In this paper, the quantity Δ is referred to as the reference node invariance threshold, or RNI threshold, for the system.

For generation or demand increments below the RNI threshold, then life is very simple. What the result in para 3.4 means is that, for such increments, marginal costs are invariant to choice of reference node: and also, marginal costs scale up, in the sense that the marginal cost for an increment ε is simply the marginal cost for a unit increment, multiplied by ε.

However, once we are dealing with increments which are above the RNI threshold for the system, both of these features break down. The example considered in the following section illustrates this, for a very simple network.

Section 4: Example of a simple transmission network.

4.1) This section illustrates the implications of the above theory with reference to a simple linear transmission network. There are 5 nodes, joined in order 1,2,3,4,5. The base inputs, and link lengths to the following node, are as follows

Transmission network specification

Node	Base input	Link length
1	60	1
2	-40	5
3	-10	5
4	20	1
5	-30	

In its base position, there is a positive flow through the system from node 1 through to node 5. It is also clear that, in terms of the definition in the preceding section, the RNI threshold for this system is 10.

4.2) Suppose node 3 is taken as the reference node. Table 1 below shows the marginal costs associated with an additional increment of generation capacity at each of the other nodes, for a range of increments. In each case, in order to ease comparison, the marginal costs have been standardised by dividing by the relevant increment: so what table 1 shows, for node a, and increment ε, is $m_{a3}(\varepsilon)/\varepsilon$, in the notation introduced in para 3.1.

Table 1: Standardised marginal costs for different increments, with reference node 3.

Increment	Node 1	Node 2	3	4	5
1	6	5	0	-5	-6
5	6	5	0	-5	-6
10	6	5	0	-5	-6
15	6	5	0	-1.67	-2.67
20	6	5	0	0	-1
25	6	5	0	1	0
30	6	5	0	1.67	0.67
35	6	5	0	2.14	1.43

Table 1 illustrates a number of interesting features:-
(i) the marginal costs are indeed invariant to scaling for increments below the RNI threshold for the system, exactly as predicted by the theory in the preceding section.
(ii) But once the RNI threshold is passed, this position rapidly changes: and the apparent marginal cost advantage of nodes 4 and 5 relative to nodes 1 and 2 quickly erodes.
(iii) It is also interesting to note that the relative cost advantage of node 5 in this example starts to erode rapidly as soon as the RNI threshold is crossed – and long before the direction of flow from node 4 to node 5 actually reverses. This illustrates how a change in the direction of flow to or from an individual node cannot be taken as the decisive indicator of when the marginal cost signals for that node start to become misleading.

4.3) Table 1 illustrates how, for a given reference node, the pattern of marginal costs can change rapidly as the transmission increment changes, once the increments involved are over the RNI threshold for the system. Importantly, the converse statement also holds, in the sense that, for a fixed increment, (above the RNI threshold), the relativities between marginal costs can vary markedly, as different reference nodes are chosen. This is illustrated in Table 2, which, for a fixed generation increment of 35, shows the scaled marginal costs which result from different choices of reference node. (Formally, for input node a and reference node b, what is shown in the table is $m_{ab}(35)/35$.)

Table 2: standardised marginal costs for different reference nodes, with increment 35.

	Inp. node 1	2	3	4	5
Ref node 1	0	-1	-1.71	0.43	-0.29
2	1	0	-0.71	1.43	0.71
3	6	5	0	2.14	1.43
4	11	10	5	0	-0.71
5	12	11	6	1	0

The table illustrates how radically both the strength of the price signals, and their direction, change as the reference node changes:-
(i) a measure of the strength of the price signal given by a particular set of marginal costs is the difference between the largest and smallest marginal cost in the given row of the table. For reference node 1, (the first row in the table), the difference between the smallest and largest marginal costs is 2.14: while for reference node 5, the corresponding difference is 12.
(ii) As regards the direction of the price signals being given, when the reference node is taken as node 1 or 2, then the highest marginal cost node is node 4, and the cheapest is node 3. When the reference node is node 4 or 5, however, the quite different signal is being given that the highest marginal cost node is node 1, and the cheapest is node 5.

4.4) While the example illustrated in this section is, of course, extremely simple, it does illustrate one key message very clearly. Once we are dealing with generation increments which exceed the RNI threshold of the system in question, then there is no single choice of reference node which will give price signals which are reliable as to either magnitude or direction.

Section 5: Implications for NG transmission pricing.

5.1) The first question that arises is: what is the RNI threshold likely to be for the NG transmission network? It seems likely that the RNI threshold is likely to be close to the smallest flow on any link in the equilibrium system. If so, it seems likely that the threshold will be smaller than the input at many generation nodes in the system.

5.2) What are the implications for the NG transmission cost model? The primary purpose of this paper is to assess whether the marginal costs calculated by the model satisfy the property claimed by NG, that the costs "provide appropriate signals for efficient economic decisions by the user." For prices to give this kind of signal to

generators, they must possess the following property. Namely that, if a generator is expanding capacity, or a new generating unit is being attached, then, (leaving aside the portion of the fixed cost of the transmission system that a new unit might attract), the change in NG's transmission costs should equal the change in the charge to the operator.
As has been seen above, for increments ε above the RNI threshold for the system, then in general $\mathrm{m}_{ax}(\varepsilon) \neq \varepsilon\,\mathrm{m}_{ax}(1)$: moreover, the difference between these two quantities may be very large – and it also now matters which particular reference node x is selected. That is, the pricing signal given to the operator, (i.e., $\varepsilon\,\mathrm{m}_{ax}(1)$), is unlikely to reflect what the overall change to NG's transmission costs will be, and hence is potentially very misleading. The implication is that locating generation capacity in line with these pricing signals is unlikely to lead to the optimal location of generation capacity.

5.3) It might be thought that, since NG make the transmission charging model available to operators to run their own network scenarios, then an operator considering a large generation increment would be expected to run the model to see what the cost implications of the scenario would be: and that this therefore gets round the above problem. But it does not avert the problem, for the following reason. What the operator is doing is working out another incremental marginal cost, (let us denote this $\dot{\mathrm{m}}_{ax}(1)$), for an adjusted network. But this adjusted network will also have its own RNI threshold: and this will probably be close to the RNI threshold for the original network. So it is likely that the generation increment will be above the RNI threshold for the adjusted network: in which case, scaling up the quantities $\dot{\mathrm{m}}_{ax}(1)$ for the adjusted network is no more likely to come close to the actual change in network transmission costs than scaling up the quantities $\mathrm{m}_{ax}(1)$ for the original network.

5.4) To determine how potentially misleading the NG prices are, it would be necessary to do empirical work on the existing network. Specifically, what is required is to investigate by how much the quantities $\mathrm{m}_{ax}(\varepsilon)$ differ from the quantities $\varepsilon\,\mathrm{m}_{ax}(1)$, for generation increments ε likely to be encountered in practice, and for different reference nodes x.
Note that, for the purpose of investigating this question, the relevant choice of ε is likely to be the largest potential generation change which could be made to the system. For example, if the choice is between putting in a number of small windfarms, or one large thermal station, it is relevant to consider an ε which equals the input at the thermal station.

5.5) What is the likely direction of any biases in the current NG transmission cost system? The current system has been characterised as one where there is basically a North to South flow. Examination of simple linear systems exhibiting North South flow will thus give some indication of the likely direction of bias when generation increments are above the RNI threshold of the system. It is not difficult to solve such linear systems algebraically: and it is clear that, for generation increments above the RNI threshold, cost estimates obtained by scaling up marginal costs based on an infinitesimal increment will tend to be relatively larger for northern generators than direct estimates of the actual change in transmission cost. Given this, it is likely that the present system involves unduly high charges for generators in the North of the system.

5.6) It is also worth noting that, as was demonstrated by the example in section 4, the scaled up unit marginal cost at a node can be giving a very misleading price signal, long before any of the flows to or from that particular node are on the point of changing direction. It is therefore not an appropriate approach to use the point where the flow to a node changes direction as a diagnostic test to establish whether or not the pricing model is accurate at that node.

5.7) The position is not so obviously worrying on the demand side. If the decision increments on demand are usually small, which could well be the case if this reflects individual consumers making marginal changes in response to price signals, then the typical demand increment may be well below the RNI threshold.
This would imply that there may well be a justification for an asymmetric approach towards connection charges, as between generation and demand. Under such an approach, marginal costs on the demand side could continue to be based on a small increment – since the relevant decision unit for most users of the system would be below the RNI threshold.

5.8) It does not appear that there is likely to be any simple complete solution to the identified problems on the generation side. The implication of the theory developed in this paper is that, once we are dealing with increments above the RNI threshold, it is not meaningful to choose one individual reference node as a basis for calculating marginal costs: and radically different marginal cost estimates could well result from different choices of reference node. In intuitive terms, this conclusion appears to make sense. What the NG theory seems to imply is that a decision could be taken about the optimal allocation of a major new generation input, without considering specifically where the extra demand to absorb the new input was going to be located, or what compensating reductions in generating capacity elsewhere might have to be made. It is clear that real world decisions could not be taken without specifically modelling such aspects.

5.9) However, while there may be no simple complete solution to the problems on the generation side, this does not in itself represent a clinching argument for keeping the status quo. The present NG charging model is relatively simple: but other simple approaches to charging are possible. Two possibilities, for example, would be a flat charging system: or marginal costs averaged over all possible reference nodes – that is $n^{-1}\sum_{x} m_{ax}(\varepsilon)/\varepsilon$, for some ε above the RNI threshold for the system. It is perfectly possible that one of these approaches, or some other approach, might have the practical advantage of simplicity, while at the same time giving less misleading price signals than the current system. Whether there is a better alternative approach is essentially an empirical question, which needs to be investigated.

5.10) In conclusion, what this paper has done is to point out potential problems which are likely to affect the NG transmission model, and which imply that the current system is very unlikely to give optimal price signals about the location of new generation capacity. What the paper has not done is to suggest an alternative – partly because there are very unlikely to be any easy answers. Nevertheless, it appears that the potential problems with the existing system are so serious that active consideration should be given to addressing these problems. In particular, the following steps should now be considered:-

a) to establish what the RNI threshold is for the present network.

b) if this is below the increment involved in real world generation decisions, to investigate empirically how misleading the price signals given by the present system are likely to be. Specifically, this would involve establishing how much the quantities $m_{ax}(\varepsilon)$ differ from the quantities $\varepsilon\, m_{ax}(1)$, for generation increments ε likely to be encountered in practice, and for different reference nodes x.

c) In the light of a) and b), and assuming simplicity is an important requirement of any eventual pricing system, to consider which potential simple model gives the least misleading price signals.

<u>Annex: proof of result in para 3.4.</u>

1) The first stage is to prove that

$$\delta_{ab}{}^{(xy)}(\varepsilon) = \delta_{ab}{}^{(xz)}(\varepsilon) + \delta_{ab}{}^{(zy)}(\varepsilon) \text{ for all a, b, x, y and z.}$$

(This is a general result, and does not depend on the conditions of the theorem holding).

Using the theory, and some of the notation, set out in section 1.2 of the NG user guide, the equilibrium flow from a to b is given by

$$e_{ab} = y_{ab}(\theta_a - \theta_b) \qquad (1)$$

where y_{ab} is the inverse of the circuit reactance between a and b,

and θ is a solution of a particular matrix equation

$$M\theta = p \qquad (2)$$

where p is the vector of power inputs and demands for the nodes in the system.

Let $v^{(xy)}(\varepsilon)$ be a vector of zeros, apart from $+\varepsilon$ in position x, and $-\varepsilon$ in position y, and let $\theta^{(xy)}(\varepsilon)$ be a solution of the equation

$$M\theta = p + v^{(xy)}(\varepsilon) \quad . \qquad (3).$$

Let $\Delta^{(xy)}(\varepsilon) = \theta^{(xy)}(\varepsilon) - \theta$, where θ is the solution of the equilibrium equation (2).

The first step is to show that there exists $\Delta^{(xy)}(\varepsilon)$ satisfying

$\Delta^{(xy)}(\varepsilon) = \Delta^{(xz)}(\varepsilon) + \Delta^{(zy)}(\varepsilon)$ for all x, y and z.

The only complication is due to the fact that M is singular, of rank (n-1). Given this, we can arbitrarily set $\theta_n = 0$ in solving the basic equation (2), and then deal with the reduced equation $\dot{M}\theta = p$, where $\dot{M}$ is the leading (n-1,n-1) submatrix of M, and θ and p are now truncated (n-1) vectors.

Equation (2) then has the solution $\theta = \dot{M}^{-1}p$,

and equation (3) has the solution $\theta^{(xy)}(\varepsilon) = \dot{M}^{-1}(p + v^{(xy)}(\varepsilon))$.

It immediately follows that

$$\Delta^{(xy)}(\varepsilon) = \dot{M}^{-1}\, v^{(xy)}(\varepsilon) \ .$$

Since $v^{(xy)}(\varepsilon) = v^{(xz)}(\varepsilon) + v^{(zy)}(\varepsilon)$, it follows that

$$\Delta^{(xy)}(\varepsilon) = \Delta^{(xz)}(\varepsilon) + \Delta^{(zy)}(\varepsilon)\,, \qquad (4)$$

and the same is true when we expand the Δ vectors to be n vectors, with 0 in the n'th position.
Going back to equation (1), it follows that

$$\delta_{ab}^{(xy)}(\varepsilon) = y_{ab}(\theta_a^{(xy)}(\varepsilon) - \theta_b^{(xy)}(\varepsilon)) - y_{ab}(\theta_a - \theta_b)$$

$$= y_{ab}(\Delta_a^{(xy)}(\varepsilon) - \Delta_b^{(xy)}(\varepsilon)) \ ,$$

From which it immediately follows, because of (4) that

$$\delta_{ab}^{(xy)}(\varepsilon) = \delta_{ab}^{(xz)}(\varepsilon) + \delta_{ab}^{(zy)}(\varepsilon) \ . \qquad (5)$$

2) If e_{ab} and $e_{ab} + \delta_{ab}^{(xy)}(\varepsilon)$ have the same sign, then the change in the absolute transmission load on link ab

$$= \left| e_{ab} + \delta_{ab}^{(xy)}(\varepsilon) \right| - \left| e_{ab} \right|$$

$$= \delta_{ab}^{(xy)}(\varepsilon) \operatorname{sgn}(e_{ab}) \ ,$$

where sgn denotes sign.

Let L_{ab} be the length of link ab. Then, if the conditions of the theorem hold, it follows that

$$m_{xy}(\varepsilon) = \sum_{a<b} L_{ab} \delta_{ab}^{(xy)}(\varepsilon) \operatorname{sgn}(e_{ab}) \qquad (6)$$

$$= \sum_{a<b} L_{ab} (\delta_{ab}^{(xz)}(\varepsilon) + \delta_{ab}^{(zy)}(\varepsilon)) \operatorname{sgn}(e_{ab})$$, by equation (5),

$$= m_{xz}(\varepsilon) + m_{zy}(\varepsilon)$$, which, by the result noted in para 3.2, establishes RNI at scale ε.

3) Finally, going back to stage (1), it is immediately apparent that

$$v^{(xy)}(\varepsilon) = \varepsilon v^{(xy)}(1) \ ,$$

From which it follows that

$$\delta_{ab}^{(xy)}(\varepsilon) = \varepsilon \delta_{ab}^{(xy)}(1) \, ;$$

Substituting into equation (6), the final part of the proof follows.