Re-Estimating Beta

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Introduction

This note provides an update on the beta estimates reported in Robertson (2018) and also considers the issue of debt-equity ratios (gearing) on the calculation. In that note I argued that in estimating equity betas:

1. that the time varying nature of beta should be acknowledged, both in the estimation method and (importantly) in forecasting future betas over the horizon relevant for regulatory control.
2. that the highest frequency data should be used without running into liquidity, bid-ask market microstructure issues.
3. that the maximum time interval of data should be used consistent with a view that there is no underlying structural shift in the process generating the equity beta (ie the correlations and variances of the underlying asset prices can be reasonably assumed fixed).

The conclusion reached was to use five or ten years of daily data and for estimation either

- realised beta (OLS) over the full sample with the estimated coefficient providing the forecast
- GARCH estimation of the full sample and calculation of the implied long run beta from the estimated second moments
- a rolling realised beta estimation where beta is estimated on daily data over a (say) quarterly interval and the resulting estimates forecast forward using (say) an AR(1) model.

Extensive estimation and simulation showed that the estimates from these approaches varied little in magnitude in statistical terms (though these slight differences may be of greater economic value). It should be emphasised that an approach of using simply the most recent data, estimating by OLS and using the

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estimate as the forecast (over a horizon that may exceed the estimation period) was not advocated (and some exploration of reasons for this will be discussed below).

This note extends and updates these comparisons to cover the more recent turmoil in equity markets, where clear changes in both variances and covariances might be expected to change the signal-noise ratio in the data and confirm or refute earlier analysis.

One aspect of regulatory requirements that was not addressed previously was the appropriateness of using equity beta for these purposes or whether asset betas correcting for firm gearing should be used. I therefore include discussion of different approaches to the adjustment that might be required in forecasting beta due to the effect of gearing.

Equity and Asset Beta

I briefly review the theoretical issues on calculating beta. In an environment in which the CAPM holds - broadly mean-variance analysis holds so representative agents have quadratic utility or asset returns belong to a location-scale family (which includes, but is not limited, to normal distributions) then the portfolio choices of utility maximising rational agents leads to a relationship such that the (expected) return on an asset i and that on the relevant market M are linked by

$$E\left(R_i\right) - R_f = \beta_i \left(E\left(R_M\right) - R_f\right) \tag{1}$$

where $\beta_i = \frac{Cov(R_i, R_M)}{Var(R_M)}$.

Thus we can write a relation between actual returns

$$R_i = \alpha + \beta_i R_M + u_i \tag{2}$$

where u_i collects the expectational errors, and, in a rational world, would be orthogonal to the information set used to form those expectations. In particular u_i will be zero mean conditional on this information $E\left(u_i \mid \Omega\right) = 0$. It is important to note that (2) is not a classical linear regression model capturing a causal relation between market and individual returns. It is an equilbrium relation derived from individual maximising behaviour.

If β_i is assumed a constant (which would follow if both $Cov\left(R_i, R_M\right)$ and $Var\left(R_M\right)$ are time invariant) then given, a time series of data on returns for equity i and market M, one can estimate β_i as

$$\hat{\beta}_i = \frac{\sum_t \left(R_{it} - \bar{R}_{it}\right)\left(R_{Mt} - \bar{R}_{Mt}\right)}{\sum_t \left(R_{Mt} - \bar{R}_{Mt}\right)^2} \tag{3}$$

wherein the numerator is an estimator of $Cov\left(R_i, R_M\right)$ and the denominator of $Var\left(R_M\right)$. A convenient way to obtain this estimate is to use standard statistical software to estimate a regression of R_{it} on R_{Mt}. It is important to note that this should not be thought of as estimating (2) as a regression equation (so worries about standard regression misspecification tests are not germane because we are not trying to get unbiased estimates of a causal regression coefficient).

If β_i is not a constant then one can still perform this calculation, but the $\hat{\beta}_i$ so obtained would be some weighted average of the (time varying) underlying β_i. The (statistical) properties of the expression in (3) then depend on the (joint) distribution of the random process $\left\{\begin{pmatrix} R_{it} \\ R_{Mt} \end{pmatrix} : t = 1, 2, 3 \ldots\right\}$.

Few practitioners believe that β_i is constant and the usual approach is to perform a rolling OLS over some short recent interval (such as two or five years) of the LS regression in (2) and use the estimated value as the forecast value. This involves an assumption that, at each estimation point, beta has varied historically (thus requiring a short recent estimation interval) but will not vary in the future forecast period. The estimation interval chosen then reflects a trade-off between having enough observations in the window to improve statistical properties whilst hoping the underlying parameter has not varied "too much" over this period. In Robertson (2019) I argued that an estimation approach that acknowledged the time varying properties of beta is required. Furthermore, in the regulatory context, given the need to forecast over a horizon of several years some understanding of the time series properties of beta is required (so the fact that rolling OLS and more sophisticated techniques might often generate similar estimates does not provide a defence of the rolling OLS approach). This leads to the suggestions outlined above. In GARCH the variances and covariances are parameterised to vary on a daily basis so that beta varies around a stable long run value. Realised beta assumes beta varies continuously and the (cumulated) values are calculated at, say, quarterly frequency using daily data, and are then forecast forward using a simple time series model. In both cases a long run of high frequency data is used.

One issue that was not addressed was that the CAPM generates an expected rate of return for risky investments that are equity financed. Some allowance needs to be made for debt financing. Since the beta of a portfolio is simply the (weighted) sum of the betas of the underlying components the standard approach for calculating beta of a levered firm is to take a weighted sum of the firm's equity and debt betas. In a straightforward way this would give the following relationship between asset and equity beta

$$\beta_{asset} = \frac{E}{E+D}\beta_{Equity} + \frac{D}{D+E}\beta_{Debt}$$

where E and D are respectively the value of the equity and debt of the firm. Debt betas are rather difficult to estimate but for a large well run regulated monopoly there are good arguments that β_{Debt} should be very low or indeed zero. And this is an assumption that is often made. In this report I calculate asset betas using debt beta values of both 0 and 0.125 for these companies for comparison.

In the Sections below I update the earlier estimates and also provide some overview of recent market events through daily beta estimates. I also discuss how gearing can be used to adjust the equity beta historically and, more importantly, the issues it causes for future forecasting.

Equity beta updates

Daily data on share prices for the utilities and all share index are used. The full dataset comprises 6154 observations covering the trading days from 01/01/1996 to 11/05/2020. Table 1 gives estimated betas (reported to 3 dp) from various estimation procedures using different sample intervals. All estimation was performed in Eviews. Standard errors are not reported because we are here interested in point estimates not inference.

Some points to note.

- The final column uses the 504 day CAPM estimate from 11th May 2020. The rolling CAPM estimates themselves are quite dispersed though at least over the past 15-20 years appear to vary around a long run stable level (see Figure 1). Using post 2000 the min and max values (of the rolling CAPM) are for NG (0.38,0.80), for PNN (-0.04,0.78) for SSE (0.20,1.11) for SVT (0.02,0.82) and for UU (0.27,0.82). The estimate of beta is thus highly dependent on the date chosen for the (here) two year window.

- The GARCH model also generates daily estimates of beta. A 504 day moving average of these daily estimates follows the 504 day rolling CAPM estimates pretty closely, lending weight to the argument above that the rolling OLS is actually calculating a weighted average of an underlying time varying beta - see Figure 2.

- The rolling OLS is subject to some abrupt changes and instability with substantial changes in the estimate from relatively small changes (often just one or two days) in the estimation interval. These graphs emphasise the importance of a time varying model for beta recognising that there are short term fluctuations around a much more stable underlying level.

- Figure 3 graphs four alternative methods of estimating the long run beta (plus the rolling and smoothed average versions). This done using the post 2000 data though the charts for ten years of data are similar. A number of features stand out

  - The rolling OLS CAPM (and smoothed daily GARCH betas) do appear to revert to the long run levels - the distribution of time above and below the long run is asymmetric possibly due to skewness in the (distribution of the) daily betas.
  - The implied longs runs using full sample OLS, the long run derived from an AR1 of 63 day realised betas, and the GARCH estimates calculated as average covariance/average variance all agree pretty closely in each case. The level calculated from the estimated GARCH coefficients tends to be lower.
  - There is a fair degree of persistence in the time varying betas. The 63 day realised betas have a first order autocorrelation around 0.4-0.5 except for NG which has very little autocorrelation (see Table 2).

- There is no indication of non-stationarity in the daily betas - on post 2000 data the ADF statistics are all around -10 (5143 obs). Note that although the GARCH assumes stationarity of the covariance and variance processes their ratio (ie beta) is not restricted.
- 63 day realised beta estimates also show stationarity - on post 2000 data (81 obs) we get ADFs are NG -8.4, PNN –5.5, SSE -5.2, SVT -5.2 and UU -6.0.

Gearing

Figure 4 shows gearing defined as net debt/enterprise value. Here we shall operate at monthly frequency. Enterprise value (EV) is available as daily data which is converted to monthly by averaging. The net debt data is only at 6 monthly intervals so is interpolated as constant within the six months. Movements within the six months of these variables are thus entirely driven by movements in EV. By and large gearing is relatively flat over the last ten years (regressions of gearing on a time trend are reported in Table 3. The trend variable is statistically insignificant for all except SSE (at the 5% level). The implied change due to trend over the 10 year period is given in Column 4 and is a 2nd or 3rd decimal place effect, again excepting SSE where the trend increase over the period is of the order of 0.14 (from an initial gearing level around 0.25). Where the trend is statistically insignificant the defensible approach would be to forecast gearing forward at the level of its historical average over the estimation period. For simplicity I use the same approach for SSE though a case could be made that the trending behaviour is better represented as discrete shifts in gearing around about 2008 and 2016 to a current level around 0.4.

Table 4 reports equity beta estimates calculated from 5 year,10 year and post-2000 samples of daily data using GARCH, realised beta (63 day pus AR1) and full sample OLS estimation. Columns (D)-(I) then calculate asset betas using an assumed debt beta of 0.125 (Columns (D),(E) and (F)) and 0 (Columns (G),(H) and (I))for comparison. Gearing is assumed constant at the average (monthly) level in the estimation period (reported in the gearing column).

Table 1: Beta Estimates

	(A)	(B)	(C)	(D)	(E)	(F)
			Full			
NG	0.515	0.609	0.610	0.609	0.623	0.632
PNN	0.334	0.453	0.415	0.441	0.419	0.508
SSE	0.472	0.545	0.575	0.542	0.603	1.056
SVT	0.444	0.555	0.495	0.518	0.518	0.543
UU	0.459	0.587	0.552	0.568	0.575	0.560
			2000			
NG	0.471	0.588	0.591	0.578	0.615	0.632
PNN	0.379	0.502	0.427	0.473	0.450	0.508
SSE	0.475	0.551	0.585	0.551	0.627	1.056
SVT	0.445	0.551	0.494	0.509	0.530	0.543
UU	0.438	0.557	0.540	0.528	0.569	0.560
			10 yrs			
NG	0.529	0.596	0.570	0.578	0.587	0.632
PNN	0.539	0.625	0.552	0.622	0.565	0.508
SSE	0.600	0.632	0.700	0.632	0.786	1.056
SVT	0.562	0.643	0.547	0.610	0.575	0.543
UU	0.541	0.621	0.553	0.610	0.573	0.560
			5 yrs			
NG	0.496	0.605	0.581	0.581	0.630	0.632
PNN	0.548	0.661	0.556	0.634	0.592	0.508
SSE	0.706	0.727	0.876	0.707	0.974	1.056
SVT	0.549	0.630	0.562	0.587	0.591	0.543
UU	0.583	0.656	0.595	0.602	0.614	0.560

Note:

1. Column (A) GARCH(1,1) beta calculated from estimated coefs
   Column (B) GARCH(1,1) average of daily betas
   Column (C) GARCH(1,1) average covariance/average variance
   Column (D) 63 day realised beta forecast forward using AR1
   Column (E) Full sample CAPM
   Column (F) Rolling window CAPM (504 days) final obs.

2. Sample periods:
   Full: 03/01/96-11/05/20 (6154 obs)
   2000: 01/01/00-11/05/20 (5144 obs)
   10 yrs: 12/05/10-11/05/20 (2527 obs)
   5 yrs: 12/05/15-11/05/20 (1265 obs)

Table 2: Realised beta models

Realised betas estimated on 63 day non-overlapping intervals.
Fitted AR1 using 81 observations (post 2000).
Long run calculated from estimated coefficients.

	Av	α	ρ	R^2	DW	$\frac{\alpha}{1-\rho}$
NG	0.58	0.54	0.01	0.00	2.0	0.58
PNN	0.47	0.26	0.45	0.21	2.1	0.47
SSE	0.54	0.31	0.44	0.18	1.8	0.55
SVT	0.51	0.26	0.48	0.23	2.1	0.51
UU	0.53	0.33	0.38	0.15	1.9	0.53

Note: These are the regressions underlying Column (D) in Table 1 (post 2000 data). Av is sample average of the dependent variable. α and ρ the estimated intercept and AR1 coefficient. DW is the Durbin-Watson test statistic for residual autocorrelation and the final column the estimated long run average value implied by the estimated coefficients.

Table 3: Gearing trend models

Regression is: gearing $= a + b \times \text{trend} + \varepsilon$

Sample period 2010M04-2020M03 (120 obs)

	$\hat{b}$	se	t-ratio	120 month effect
NG	-0.00026	0.00014	-1.90	-0.016
PNN	-0.00007	0.00009	-0.80	-0.004
SSE	0.00114	0.00014	8.08	0.137
SVT	0.00004	0.00009	0.48	0.003
UU	0.00005	0.00008	0.54	0.003

Note: Column (2) is estimated coefficient on (monthly) trend. Column (3) estimated standard error. Column (4) the t-ratio and Column (5) the implied ten year change in gearing from the estimated trend.

Table 4: Equity and Asset betas - different estimations and sample periods

Estimation period post 2000

	(A)	(B)	(C)	gearing	(D)	(E)	(F)	(G)	(H)	(I)
NG	0.591	0.578	0.615	0.482	0.366	0.359	0.379	0.306	0.299	0.319
PNN	0.427	0.473	0.450	0.477	0.283	0.307	0.295	0.223	0.247	0.235
SSE	0.585	0.551	0.627	0.257	0.467	0.442	0.498	0.435	0.409	0.466
SVT	0.494	0.509	0.530	0.517	0.303	0.310	0.321	0.238	0.246	0.256
UU	0.540	0.528	0.569	0.519	0.325	0.319	0.339	0.260	0.254	0.274

Estimation period last 10 years

	(A)	(B)	(C)	gearing	(D)	(E)	(F)	(G)	(H)	(I)
NG	0.570	0.578	0.587	0.452	0.369	0.373	0.378	0.312	0.317	0.322
PNN	0.552	0.622	0.565	0.448	0.360	0.399	0.368	0.304	0.343	0.312
SSE	0.700	0.632	0.786	0.307	0.524	0.477	0.583	0.485	0.438	0.545
SVT	0.547	0.610	0.575	0.525	0.325	0.356	0.339	0.260	0.290	0.273
UU	0.553	0.610	0.573	0.558	0.314	0.339	0.323	0.244	0.270	0.253

Estimation period last 5 years

	(A)	(B)	(C)	gearing	(D)	(E)	(F)	(G)	(H)	(I)
NG	0.581	0.581	0.630	0.443	0.379	0.379	0.406	0.323	0.324	0.351
PNN	0.556	0.634	0.592	0.445	0.364	0.408	0.384	0.309	0.352	0.328
SSE	0.876	0.707	0.974	0.341	0.620	0.509	0.684	0.578	0.466	0.642
SVT	0.562	0.587	0.591	0.523	0.334	0.345	0.347	0.268	0.280	0.282
UU	0.595	0.602	0.614	0.558	0.333	0.336	0.341	0.263	0.266	0.271

Note: For each sample period columns (A), (B) and (C) are the equity betas estimated using

- Column (A) GARCH average covar/average var
- Column (B) OLS 63 day realised beta forecast by AR1
- Column (C) OLS Full sample

Columns (D), (E) and (F) are then asset betas estimated using the equity betas of Columns (A), (B) and (C), respectively, using the estimated gearing level in the column headed gearing and an assumed debt beta of 0.125.

Columns (G), (H) and (I) are then asset betas estimated using the equity betas of Columns (A), (B) and (C), respectively, using the estimated gearing level in the column headed gearing and an assumed debt beta of 0.

Figure 1 Rolling (504 day) CAPM estimates

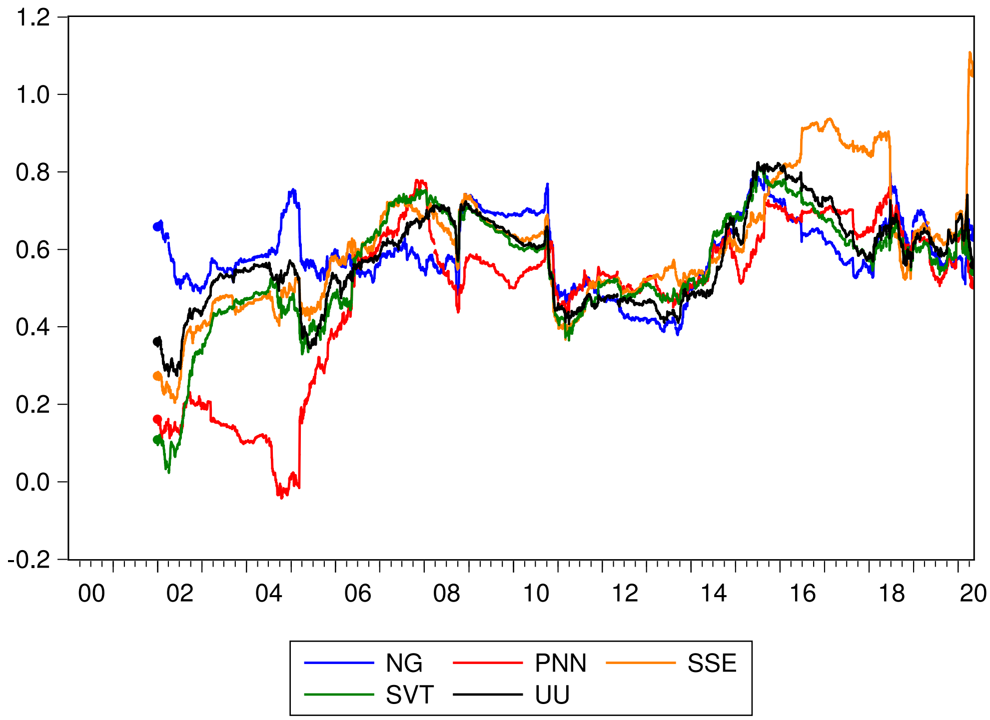


Figure 2: Rolling CAPM and smoothed daily GARCH estimates post 2000 data

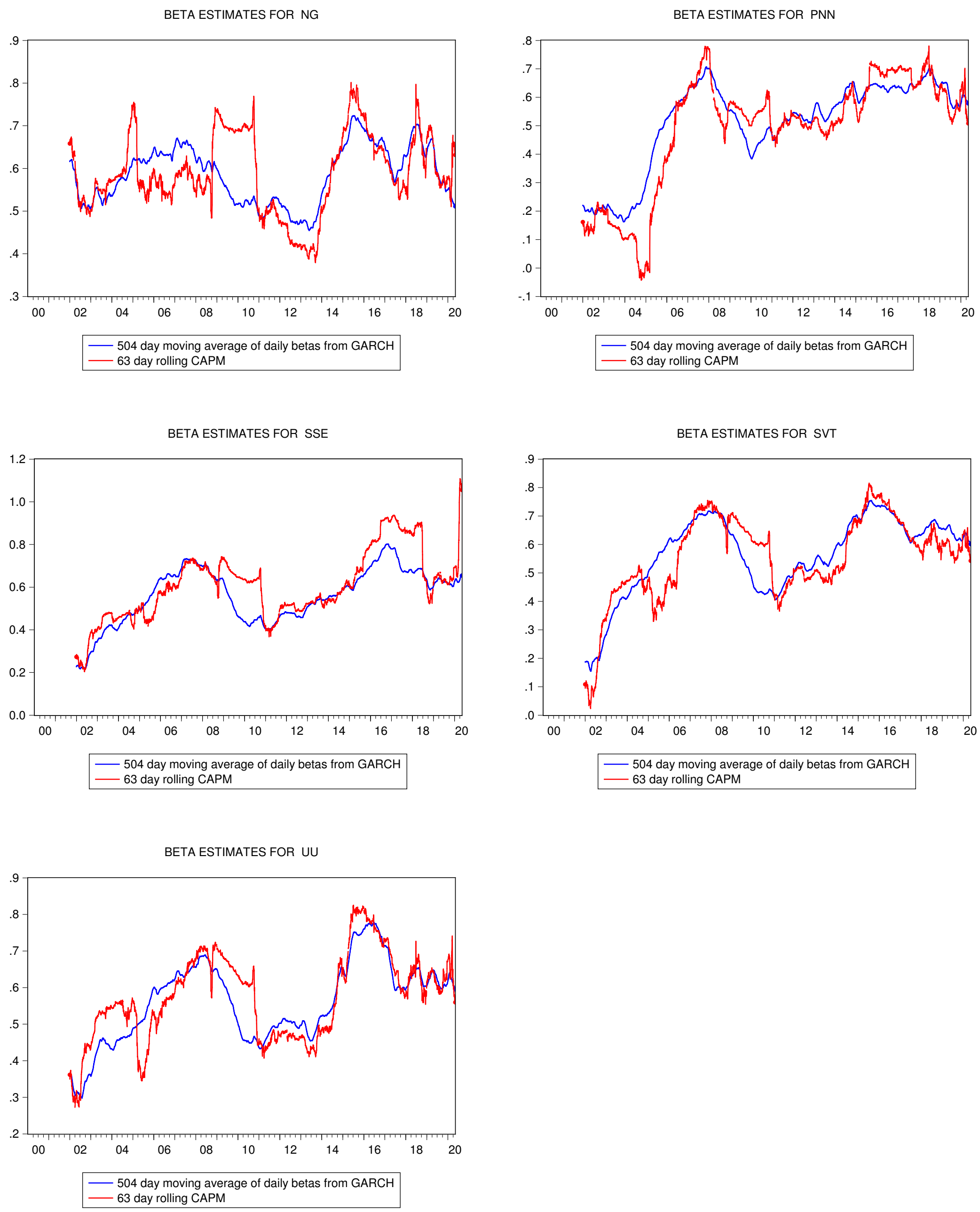
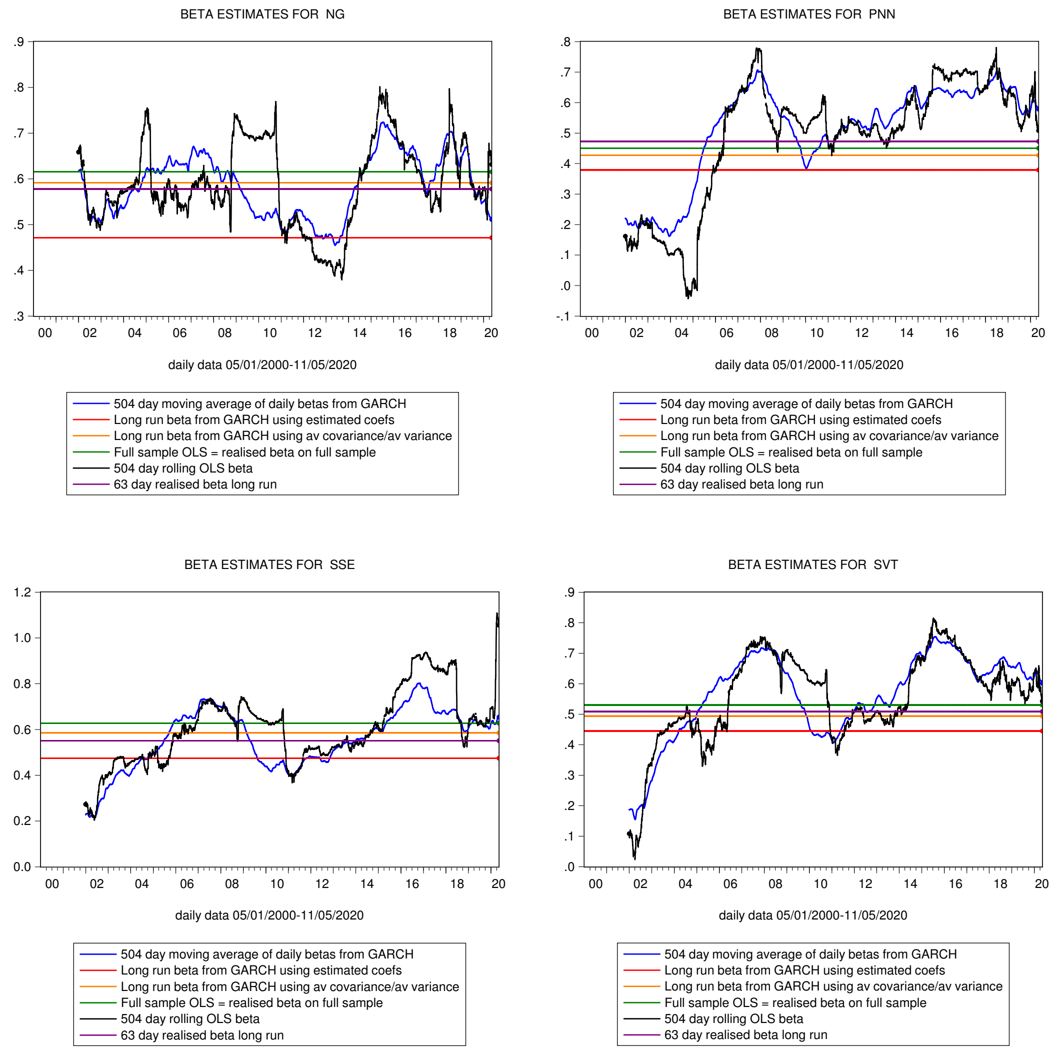
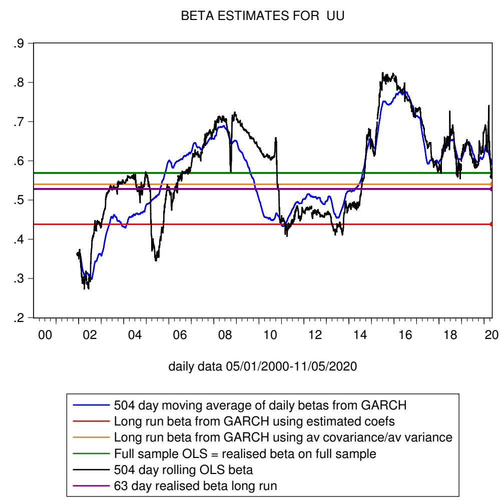


Figure 3: Beta estimates




BETA ESTIMATES FOR UU
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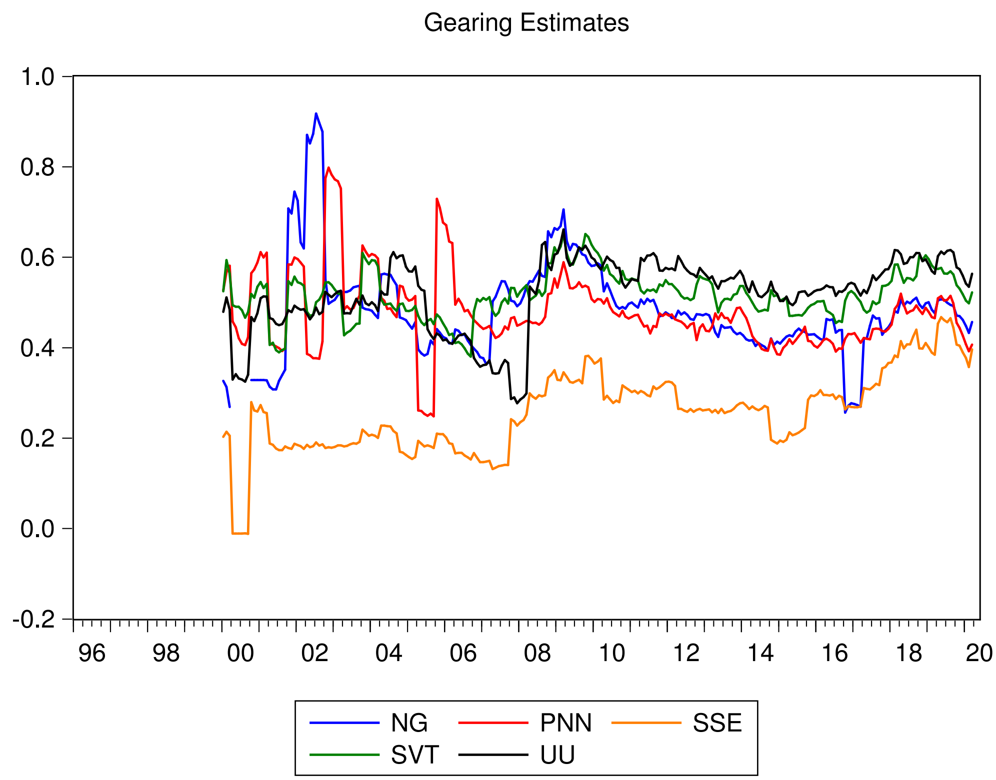
daily data 05/01/2000-11/05/2020
504 day moving average of daily betas from GARCH
Long run beta from GARCH using estimated coefs
Long run beta from GARCH using av covariance/av variance
Full sample OLS = realised beta on full sample
504 day rolling OLS beta
63 day realised beta long run

Figure 4 Gearing



Appendix I: Recent events

One advantage of the GARCH approach is that the method returns daily estimates of $Cov\left(R_i, R_M\right)$, $Var\left(R_i\right)$, $Var\left(R_M\right)$ and $\beta_i = \frac{Cov(R_i,R_M)}{Var(R_M)}$. It is interesting to view the recent market movements with the Covid-19 events through these estimates.

What becomes clear is that all utilities showed a spike in beta in December 2019. Both the covariances and market volatility rise slightly at this time but the proportionate rise in covariances is much larger giving a rapid increase in beta. Betas then revert to a more usual level through early 2020 except for SSE which sees another spike in beta. The rapid increases in the covariances and in market volatility during the Covid crisis are clearly visible in the most recent data, but for NG, PNN, SVT and UU the ratio has stayed remarkably stable over this time. By the very end of the sample even SSE has reverted to its more usual level. The dramatic spike visible in the rolling CAPM for SSE (Figure 1) looks unlikely to persist.

Figure A1: Daily cov, var and beta estimates

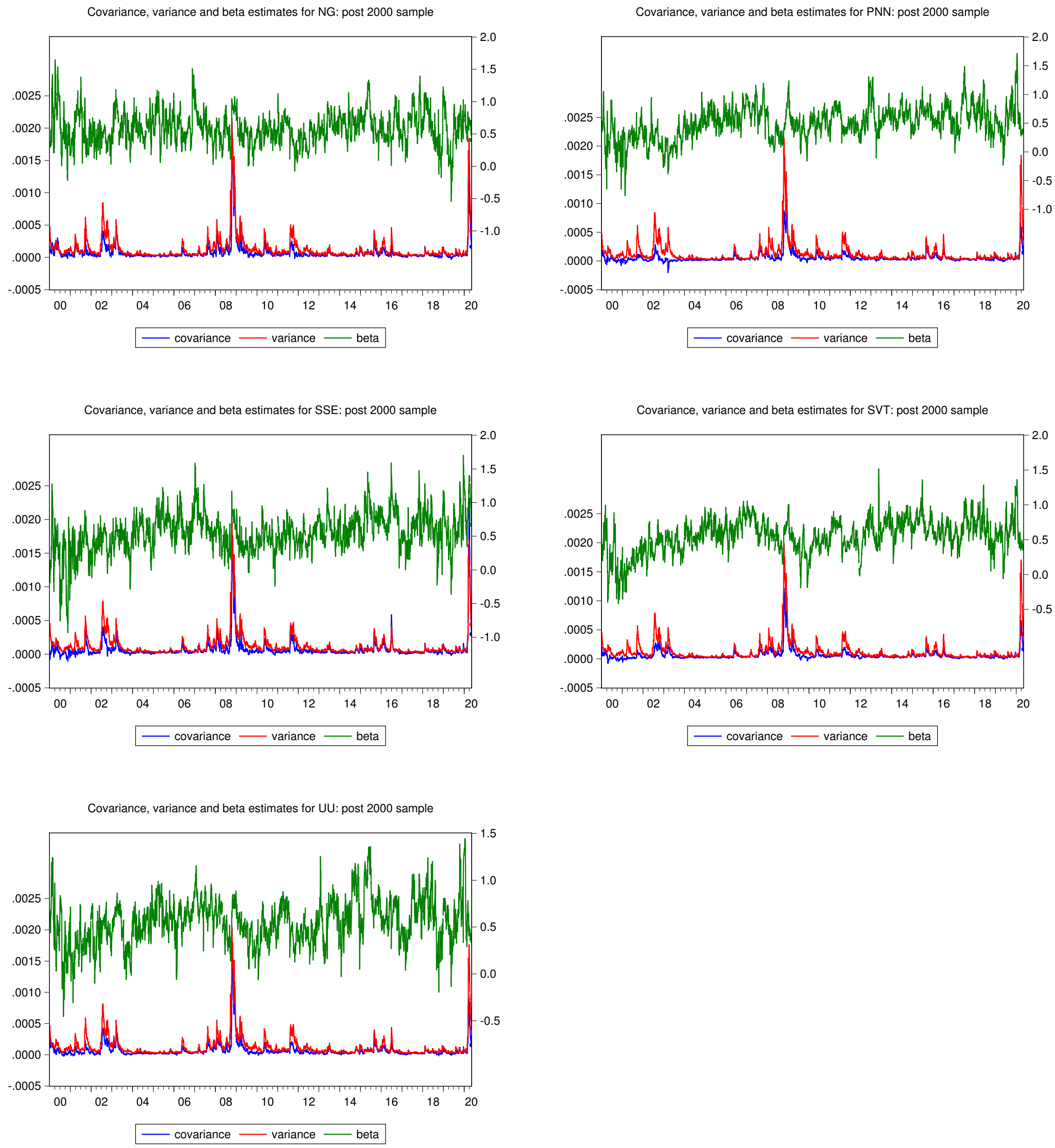
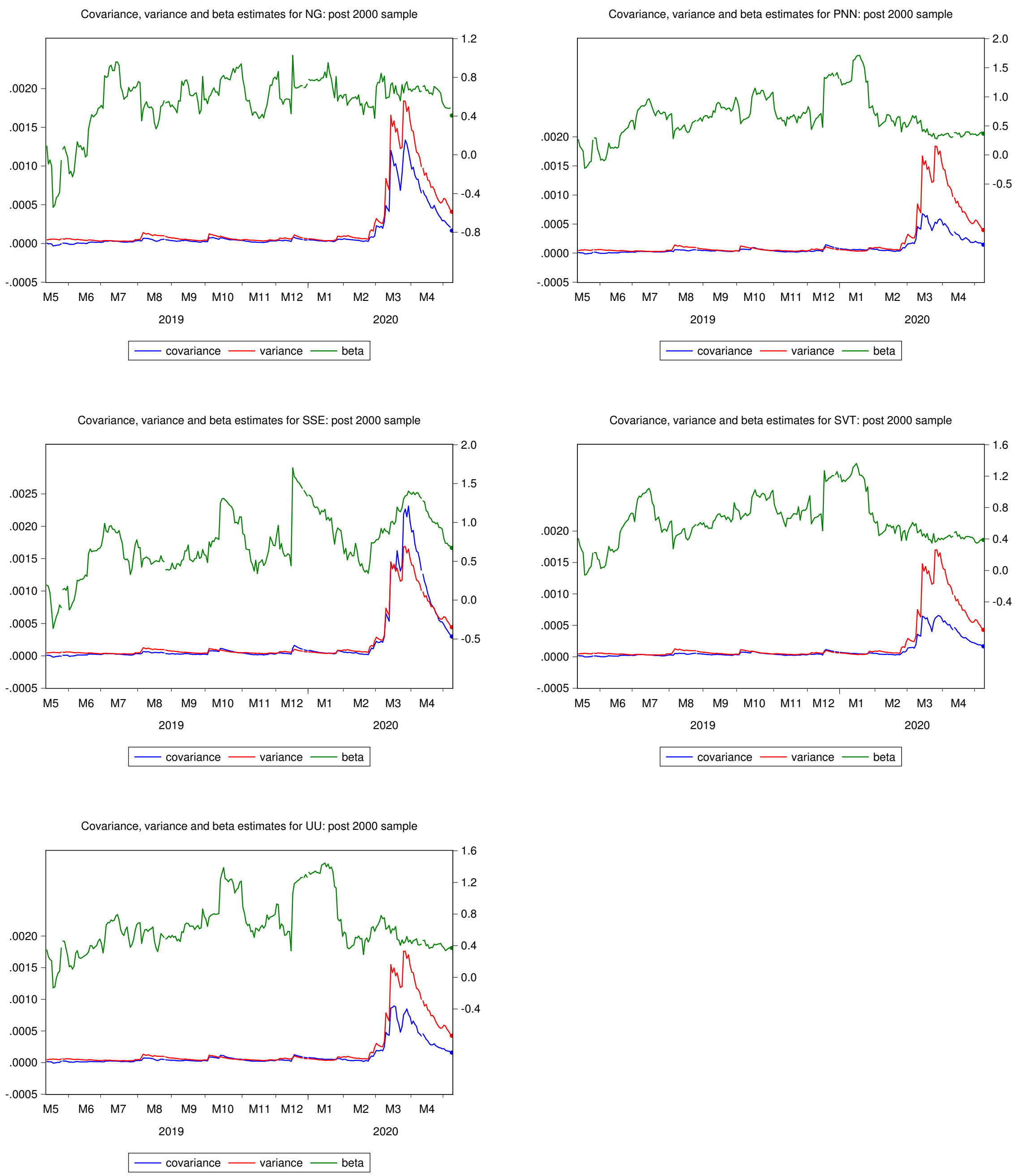


Figure A2: Recent Daily Movements



Appendix II Some issues with OLS and time varying beta

If we start from the assumption that equity betas are time varying then the expected return model for equity i is

$$R_{it} = \alpha + \beta_{it} R_{Mt} + \xi_{it}$$

where ξ_{it} captures the expectational errors so should be orthogonal to everything dated earlier. We shall assume that β_{it} is time varying around a constant mean.

Dropping the i subscript for clarity gives

$$R_t = \alpha + \beta_t R_{Mt} + \xi_t$$

Which we can rewrite as

$$R_t = \alpha + \beta R_{Mt} + (\beta_t - \beta) R_{Mt} + \xi_t$$

where β is the average of the β_t over the sample period. This gives the CAPM regression in the case of an underlying time varying beta.

Two issues arise in using OLS in these circumstances:

Firstly letting $u_t = (\beta_t - \beta) R_{Mt} + \xi_t$ and we get the CAPM regression is least squares on

$$R_t = \alpha + \beta R_{Mt} + u_{it}$$

over some sample period $T = 1, \ldots, T$. The OLS estimate is given by

$$\hat{\beta}_i = \frac{\frac{1}{T-1} \sum \left(R_{it} - \bar{R}_{it}\right) \left(R_{Mt} - \bar{R}_{Mt}\right)}{\frac{1}{T-1} \sum \left(R_{Mt} - \bar{R}_{Mt}\right)^2}$$

where the bars denote average over the sample period. It is useful to manipulate this as follows

$$\begin{aligned}
\hat{\beta}_i = \frac{Cov\left(R_{it}, R_{Mt}\right)}{Var\left(R_{Mt}\right)} &= \frac{Cov\left(\alpha + \beta R_{Mt} + u_t, R_{Mt}\right)}{Var\left(R_{Mt}\right)} \\
&= \frac{Cov\left(\alpha, R_{Mt}\right)}{Var\left(R_{Mt}\right)} + \frac{Cov\left(\beta R_{Mt}, R_{Mt}\right)}{Var\left(R_{Mt}\right)} + \frac{Cov\left(u_t, R_{Mt}\right)}{Var\left(R_{Mt}\right)}
\end{aligned}$$

so the large sample properties of the OLS estimate are given by the behaviour of these three terms. The first is heading towards zero, and the second is simply β. The third measures comovement between $u_t = (\beta_t - \beta) R_{Mt} + \xi_t$ and R_{Mt}. Typically in a random coefficients model any variation in the slope coefficient is assumed independent of the regressor so that the compound error (u_t) is then conditionally independent of the regressor and in these circumstances OLS returns the average (of the random coefficient).

However if there is comovement then this final term need not be zero, that is if the daily movements in beta have some systematic relation to market returns (or volatility) then this will get picked up by the OLS estimator.

We can dig into $\frac{Cov(u_t,R_{Mt})}{Var(R_{Mt})}$ a bit more. The sample value of the numerator is

$$\frac{1}{T-1}\sum\left(R_{Mt}-\bar{R}_{Mt}\right)\left(u_t-\bar{u}_t\right)=\frac{1}{T-1}\left(\sum R_{Mt}u_t-T\bar{R}_{Mt}\bar{u}_t\right)$$

where $u_t=\left(\beta_t-\beta\right)R_{Mt}+\xi_t$. So

$$\begin{aligned}\frac{1}{T-1}\left(\sum R_{Mt}u_t-T\bar{R}_{Mt}\bar{u}_t\right)&=\frac{1}{T-1}\sum R_{Mt}\left(\left(\beta_t-\beta\right)R_{Mt}+\xi_t\right)-\frac{T}{T-1}\bar{R}_{Mt}\overline{\left(\left(\beta_t-\beta\right)R_{Mt}+\xi_t\right)}\\&=\frac{1}{T-1}\sum\left(\beta_t-\beta\right)R_{Mt}^2+\frac{1}{T-1}\sum R_{Mt}\xi_t\\&-\frac{T}{T-1}\bar{R}_{Mt}\bar{\xi}_t-\frac{T}{T-1}\bar{R}_{Mt}\overline{\left(\left(\beta_t-\beta\right)R_{Mt}\right)}\end{aligned}$$

If we assume ξ_t is independent of the vector R_{Mt} (note this requires ξ_t unrelated to both past and *future* R_{Mt}) and zero mean then the second and third terms have *plim* zero. This leaves

$$\frac{1}{T-1}\sum\left(\beta_t-\beta\right)R_{Mt}^2-\frac{T}{T-1}\bar{R}_{Mt}\overline{\left(\left(\beta_t-\beta\right)R_{Mt}\right)}$$

Now if $\beta_t-\beta$ were independent of R_{Mt} then both of these would also head towards zero (given the assumption that the long run average of the β_t exists).

But if the movements of the daily beta β_t around its long run average β are systematically related to market volatility at the daily frequency then it is difficult to see why this expression should have zero limit.

A second problem that arises is that the parameter of interest β is actually the ratio of a covariance to a variance. In this situation it is not clear that the average value of beta is the correct object to estimate. Depending on the assumptions we make about the source of the time variation in β the OLS approach may be misleading (this point is also made in the May 2019 OFGEM report: *RIIO-2 Sector Specific Methodology Decision – Finance*) .

If beta is time varying then it must be that the covariances and/or the variances are also time varying. In the previous report I argued that one needs to specify a model for how these evolve over time. In particular if $Cov\left(R_{it},R_{Mt}\right)$ and $Var\left(R_{Mt}\right)$ vary about some long run stationary level then the long run beta is the ratio of these two long run levels and this is not the same as the average of the daily betas due to a Jensen's Inequality effect (that is the average of the ratio is not the same as the ratio of the averages). The GARCH model proposes a particular (stationary) structure for the time evolution of both $Cov\left(R_{it},R_{Mt}\right)$ and $Var\left(R_{Mt}\right)$ around their long run stationary levels. If we call these $Cov^*\left(R_{it},R_{Mt}\right)$ and $Var^*\left(R_{Mt}\right)$ and it is the ratio of these two we seek. Taking the average of daily betas (which is what OLS is doing in estimating β above) is not quite correct because there's a Jensen's inequality term. That is if β_i is the average of the daily betas then

$$E\left(\beta_i\right)=E\left(\frac{1}{T}\sum\left(\frac{Cov_t\left(R_{it},R_{Mt}\right)}{Var_t\left(R_{Mt}\right)}\right)\right)\neq\frac{E\left(\frac{1}{T}\sum Cov_t\left(R_{it},R_{Mt}\right)\right)}{E\left(\frac{1}{T}\sum Var_t\left(R_{Mt}\right)\right)}=\frac{Cov^*\left(R_{it},R_{Mt}\right)}{Var^*\left(R_{Mt}\right)}=\beta_{i,LR}$$

As argued above OLS provides an estimate of the average of the daily betas and so does not necessarily provide an unbiased estimate of the long run beta.